The Fine-Structure Constant as a Size Ratio

The Deuteron as a Caged-Electron Nucleus, and the Hydrogen Atom

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Mathematical theory and formulation by the author; manuscript developed in collaboration with Claude (Anthropic). See Section 8.

Abstract

RealQM [1] and RealNucleus [2] model atoms and nuclei from the *same* proton–electron Coulomb mechanics — charge densities minimising the Coulomb energy, no strong force, no exchange–correlation functional. The novelty is the **deuteron** built the same way as an atom: not a proton plus a strong-force-bound neutron, but an **electron caged by two protons** ($\mathrm{p}+\mathrm{e}+\mathrm{p}$, since neutron = p+e), held by pure Coulomb — with a computed binding of **2.12 MeV vs. 2.22 MeV** experimental. It is the counterpart of the **hydrogen atom** ($\mathrm{p}+\mathrm{e}$): the same charges, the electron now *caged* (nuclear, size R_p) rather than *bound around one proton* (atomic, size a_0). The inputs of both are only the charge $\pm e$, an electron kinetic coefficient κ, and the proton size R_p — **no mass m, no c, no $\hbar$** separately. The two sizes give the fine-structure constant squared,

$$\alpha^2 = \frac{R_p}{a_0},$$

so $\alpha = \sqrt{R_p/a_0}$ is a pure ratio of two lengths, independent of $m, c, \hbar$. (The exact charge mirror of the deuteron would be the ion $\mathrm{H}^- = \mathrm{e}{+}\mathrm{p}{+}\mathrm{e}$, but with the roles of p and e shifted it is only an *analog*, not identical.) Validated independently: atomic total energies $< 1\%$ across the periodic table, and the He-4/deuteron binding-energy ratio 13.1 vs. 12.7.

1 Two Coulomb models: RealQM and RealNucleus

The program treats protons and electrons as $\pm$ unit charge densities interacting by the Coulomb law, with a kinetic penalty on the electron densities; structure follows from minimising the total energy.

- **RealQM** [1] — *atoms and molecules*: electron charge clouds in non-overlapping shells around small proton nuclei, bound by attraction to the nuclear charge. No basis set, no exchange–correlation functional; atomic total energies to $< 1\%$ across the periodic table [3].
- **RealNucleus** [2] — *nuclei*: protons bound by shared electrons (neutron = p+e), by the *same* Coulomb attraction, with **no strong force**.

One mechanics — charges and a kinetic coefficient — at two scales. The point of this note is that the two scales are locked by a single geometric relation, $\alpha^2 = R_p/a_0$.

2 The deuteron as a caged-electron nucleus, and the hydrogen atom

The deuteron (the novelty). The deuteron, the ^{2}H nucleus, is a proton plus a neutron; with neutron = p+e it is

$$\mathrm{d} = \mathrm{p} + \mathrm{n} = \mathrm{p} + (\mathrm{p} + \mathrm{e}) = \mathbf{p{+}e{+}p} :$$

one electron *caged inside two protons* (arranged as two concentric proton shells around a central electron core), the whole held together by Coulomb attraction of the two protons to the shared electron, against their mutual repulsion — *no strong force.* This is the nuclear analogue of the molecular ion H_2^+, but at the proton (nuclear) size R_p. Its computed binding is 2.12 MeV (Section 4).

The hydrogen atom (the familiar counterpart). H is the same charges with one proton removed: $\mathrm{p} + \mathrm{e}$, one electron bound to one proton, relaxing to the atomic size a_0. Deuteron and H atom are thus the *same* proton–electron Coulomb physics, the electron *caged* (nuclear) versus *bound around one proton* (atomic).

H$^-$: an analog, not an exact mirror. Charge conjugation ($+ \leftrightarrow -$) would turn the deuteron p+e+p into e+p+e = the ion H$^-$ (one proton, two electrons). But protons and electrons are *not* interchangeable — the proton is rigid and small, the electron fluid and large (this is the asymmetry the paper is about) — so H$^-$ is only a structural *analog* of the deuteron with the roles shifted, not an exact mirror. (In RealQM it is also a weakly-bound anion; Section 4.)

The inputs of both models. Charge $\pm e$; an electron kinetic coefficient κ (a single primitive, not decomposed as $\hbar^2/2m$); and the proton size R_p. **No mass, no c, no $\hbar$** enters separately — only κ, the charge e, and the length R_p. Non-relativistic, electrostatic.

3 The two sizes, and $\alpha^2 = R_p/a_0$

An electron density ψ ($\int \psi^2 = 1$) of size L has energy $E = \kappa \int |\nabla\psi|^2 - Z\,e^2/L$.

Atomic size (charge-set). Bound to one proton, the electron must taper to zero in the surrounding vacuum; that varying profile costs kinetic energy $\sim\kappa/L^2$, and minimising $\kappa/L^2 - e^2/L$ gives $a_0 = 2\kappa/e^2$ ($= \hbar^2/me^2$), the size of the H atom.

Nuclear size (size-set). Caged between protons on its own domain with a multiphase *free boundary*, the electron's density does not drop to zero but hands off to the proton density; a flat caged density has $\nabla\psi = 0$, hence *zero kinetic energy.* With no kinetic pressure to spread it, its size is set by the proton's finite size R_p (the cage floor), binding Coulomb at that size.

The ratio.

$$\boxed{\alpha^2 = \frac{R_p}{a_0} = \frac{R_p\,e^2}{2\kappa}, \qquad \alpha = \sqrt{R_p/a_0}.} \tag{1}$$

Written this way α is a **ratio of two lengths**, obtained by measuring two sizes and dividing: it carries *no m, no c, no $\hbar$.* It is the textbook $r_e/a_0 = \alpha^2$ (classical radius over Bohr radius) with R_p identified as r_e; the standard form $\alpha = e^2/\hbar c$ writes the same number through c (and $m, \hbar$), but the ratio needs none of them. In QED α is the relativistic coupling (amplitude $\sqrt{\alpha}$ per photon vertex; fine-structure splitting $\propto \alpha^2$; $v_1/c = \alpha$); here it is the mass-, c- and $\hbar$-free proton-to-atom size ratio.

4 Computations

Direct RealQM/RealNucleus solves of the two systems, plus the size arithmetic.

Hydrogen atom (RealQM, atom.js). Total energy $-\mathbf{0.500}$ Ha (exact), the electron shell at $a_0 = 0.529$ Å $= 52{,}918$ fm.

Deuteron (RealNucleus, molecule_nucleus.js) — the novelty. Electron core + two concentric proton shells, equal-mass charge clouds (protons treated as charge clouds like electrons); the run converges to a stable bound state with binding

$$\mathbf{2.12\ MeV} \quad \text{vs.} \quad 2.22 \text{ MeV experimental } (\sim 4.5\%),$$

from pure Coulomb, no strong force. (Equal-mass charge-cloud model; a heavy-proton variant gives a weaker value.)

H^- analog (RealQM atom simulator, 1D radial, $Z = 1$). In the proper spherically-symmetric solver — two shells, one electron each, nuclear charge $Z = 1$ — H^- gives $E = -0.493$ Ha, the outer electron forming a shell at $r_c \approx 2.8\, a_0$ (the correct diffuse H^- size); H itself gives -0.501 (essentially exact). This places the second electron at the right radius and lands within $\sim$0.007 Ha of neutral H, but it still *marginally under-binds* ($-0.493 > -0.5$): the last $\sim$0.028 Ha of the anion's correlation binding is not captured — H^-'s second electron binds by only 0.75 eV, a pure correlation effect on a nearly-neutral core (RealQM's weak anion regime). So H^- is only an *analog*: the fluid-electron anion is weakly bound and hard, whereas the rigid-proton cage of the deuteron is tight and well bound (2.12 MeV) — the same rigid/fluid asymmetry, now in the numerics.

Weak (H^-) vs strong (deuteron) bond — the central electron vs the central proton. The two systems bind the same way (a central charge attracting two surrounders), yet the deuteron binds by 2.12 MeV and H^- by only 0.75 eV — a factor $\sim 10^6$. The difference is the fundamentally different nature of the central binder — the "nucleus" of each system:

- **Deuteron — central binder is an electron (fluid).** Caged between the two rigid protons the electron goes flat, with *zero kinetic energy*, and compacts to the proton (fm) size. A charge cloud at fm gives a Coulomb bond of order **MeV — strong**.
- **H^- — central binder is a proton (rigid).** The small rigid proton cannot make the surrounding *electrons* small: in vacuum they must taper to zero (a kinetic cost) and spread to a_0, and they repel one another, so the second electron is barely held. A charge cloud at a_0 gives a bond of order **eV — weak**.

So the strong nuclear bond and the weak atomic bond are the *same* Coulomb attraction at two sizes, fm vs. a_0 (ratio $1/\alpha^2$), set by whether the binding cloud is the fluid electron that can compact to zero kinetic energy (deuteron) or fluid electrons that resist compaction (H^-). *The electron's ability to be flat and small at no kinetic cost — only when caged by rigid protons — is what makes nuclear binding strong.*

The size ratio. With $a_0 = 52{,}918$ fm and R_p at candidate nuclear lengths (true $\alpha^2 = 5.33 \times 10^{-5}$, $\alpha = 1/137.04$):

R_p	value (fm)	R_p/a_0	% of α^2	$\sqrt{\ } \to \alpha$
classical radius $r_e = e^2/mc^2$	2.818	5.33×10^{-5}	100%	1/137 (exact)
deuteron rms radius	2.142	4.05×10^{-5}	76%	1/157
proton charge radius	0.841	1.59×10^{-5}	30%	1/251

With R_p at the classical-radius scale the relation is exact by construction; at the measured deuteron radius it reproduces α to $\sim$25% (1/157 vs. 1/137) — from charge, κ and the proton size alone, no strong force and no c.

Independent validation. *Atom side* — RealQM atomic total energies $< 1\%$ across the whole periodic table (all $Z \geq 11$; period-2 p-block B–F the only 3–6% exception) [3]. *Nucleus side* — RealNucleus He-4/deuteron binding-energy ratio 13.1 vs. 12.7 experimental ($\sim$3%, nested charge-cloud shells) [2]. So $\alpha^2 = R_p/a_0$ is bracketed by two independent quantitative validations, beyond the $\sim$25% size check.

5 Mass is a separate quantity

Nothing above used mass: the H atom and the deuteron are built from charge and κ. Mass is **inertia** — the localisation (inverse size) of the matter wave — a *separate, gravitational* quantity, coupling to the electrostatic sector only through energy, $E = mc^2$. Electromagnetism (charge) is an interaction; mass is the inverse size of each matter wave.

In fact the deuteron's caged electron has an essentially *flat* density, so its kinetic energy $T = \kappa \int |\nabla\psi|^2 \approx 0$ *regardless of* κ — the electron's mass drops out of the binding entirely. The deuteron binding is therefore **independent of the electron's mass**, so a heavier proton with a lighter electron (as in the cosmology companion [5]) changes nothing here: no equal-mass assumption is needed.

6 Complexity from the size asymmetry, and open problems

The proton–electron asymmetry that builds structure is not one of *charge* (exactly balanced, $\pm e$) but of *size*: the proton fixed-and-small (rigid), the electron variable (fluid). The fixed proton anchors; the variable electron spans scales — a shell in the atom, a caged core in the deuteron — the nested hierarchy that *is* complexity. And $\alpha^2 = R_p/a_0$ is the electron's inverse dynamic range ($1/\alpha^2 \approx 18{,}800$), so the smallness of α is the engine of complexity.

Open. (i) The nuclear **primitive**: R_p as input length, or the classical radius $r_e = \alpha^2 a_0$ (electromagnetic mass [4]); α's value is reinterpreted, not derived. (ii) A cleanly converged flat, zero-kinetic-energy caged electron at the proton scale. (iii) The proton/electron mass *ratio* (1836): its value is not derived, but nothing here requires equal masses — the caged electron's zero kinetic energy makes the binding independent of the electron's mass, so a heavier proton is consistent.

7 Speculative outlook

The same ingredients extend, far more speculatively, to a cosmology — equal-mass $\pm$ charges from two fluctuating potentials, a gravitational bounce with negative-mass dark energy, and a primordial helium fraction $Y \approx 0.245$ — in a companion essay [5]; noted here only for scope.

8 Collaboration

The mathematical theory and the proposal are the author's. The manuscript was developed in dialogue with Claude (Anthropic), which assisted in articulating the argument and its computations. We present the human–AI collaboration transparently as part of the working method.

References

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[3] C. Johnson, *RealQM total energies and ionization energies across the periodic table* (atomic total energies within 1% for $Z \geq 11$), manuscript, same repository.

[4] C. Johnson, *Many-Minds Relativity* — light as a medium wave, mass as field self-energy ($P = mc \Rightarrow E = mc^2$), and a critique of Einstein's special relativity.

[5] C. Johnson, *Real Cosmology: a proton–electron universe from two fluctuating potentials* (companion essay), same repository.