The Spinor Residue: $g = 2$ and Stern–Gerlach without Relativity

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Abstract

The charges-as-configurations reading of RealQM removes relativity and gives the electron's magnetic moment as a quantised current-winding: an orbital winding $e^{im\phi}$ carries $\mu_z = -m\,\mu_B$ with gyromagnetic factor $g = 1$. The key is that "spin" bundles two logically distinct things: the fermionic *exclusion*, which RealQM realises geometrically by non-overlapping domains with no spin variable (and which accounts for the periodic table, shells, and bonding), and the two-valued *magnetic* moment, which it does not yet carry. This leaves exactly one residue — the electron's *spin* moment has $g = 2$, twice the orbital value per unit angular momentum — and one experiment that RealQM in its present form cannot reproduce: *Stern–Gerlach*, whose two-spot splitting is the signature of two-valued spin $\frac{1}{2}$. We make the residue precise, and argue that it is *not* a demand for relativity. By Lévy-Leblond's theorem $g = 2$ follows from a first-order (linearised), *non-relativistic* spinor wave equation; the "2" is the SU(2) double cover of the rotation group, a geometric fact, not a consequence of Lorentz invariance. Concretely, writing the kinetic energy with $\boldsymbol{\sigma}\cdot\mathbf{p}$ rather than p^2 produces, in a field, the term $-q\,\boldsymbol{\sigma}\cdot\mathbf{B}$ — the $g = 2$ Zeeman coupling — automatically. The target for RealQM is therefore identified without importing relativity: equip the non-overlapping domains with the $\pm\frac{1}{2}$ spinor label the base theory already suggests, implemented as a $\boldsymbol{\sigma}\cdot\mathbf{p}$ structure, so that the moment becomes two-valued ($\mu_z = \pm\mu_B$) and Stern–Gerlach's two spots and $g = 2$ follow together. We also observe that spin's *mechanical* content is already native: a RealQM moment is a real circulating charge, so it carries real angular momentum by construction, and the Einstein–de Haas effect (a magnetised body physically rotating as its moments align) comes for free rather than as an obstacle. Nothing called "spin" need therefore be *posited* in RealQM — exclusion is geometry, angular momentum is real charge motion — and the residue reduces to a single geometric *number*, the SU(2) factor $g = 2$ (two-valued, no middle), to be derived from configuration geometry. We are explicit that this is a *target*, not a completed derivation: what is established is that the residue is geometric and reachable, not relativistic.

1 The one residue

In the current-bearing form of RealQM the magnetic moment is that of the charge current, $\boldsymbol{\mu} = \frac{1}{2}\int \mathbf{r}\times\mathbf{J}$, and an orbital phase winding $e^{im\phi}$ gives the quantised value $\mu_z = -m\,\mu_B$ [1, 2]. This is gyromagnetic factor $g = 1$: one unit of orbital angular momentum, $L_z = m\hbar$, yields m Bohr magnetons. The charges-as-configurations picture [3] builds on this to remove any need for

relativity: sizes and moments are properties of Coulomb configurations, a "free" electron is only a weakly-confined one, and the winding it can afford gives μ_B with no rest-energy scale invoked.

Exactly one thing is left unaccounted. The electron's *spin* moment is not $g = 1$ but $g = 2$: a spin of only $\hbar/2$ produces a *full* Bohr magneton, twice what an orbital current of the same angular momentum would give. An orbital winding cannot supply this factor of two; it is the signature of spin, and RealQM has, so far, no spin-$\frac{1}{2}$ moment. This is the residue.

2 What spin is: exclusion versus magnetism

That the residue is so small has a reason worth stating in its own right, because it clarifies what RealQM does and does not owe to spin. The word "spin" bundles together two *logically distinct* things:

1. **Statistics-spin** — the fermionic *exclusion*: two electrons to an orbital, antisymmetry, the Pauli principle, shell structure, the periodic table.

2. **Magnetic-spin** — the two-valued *magnetic moment*: $\mu_z = \pm\mu_B$, $g = 2$, the property that splits a beam.

The spin–statistics theorem welds them ("half-integer spin ⇒ fermion"), so in the standard formulation they appear as one property. They are not, and RealQM separates them.

Statistics-spin is geometry, not spin. Everything RealQM reproduces — atoms, the periodic table, shells, bonding, reactions, condensed phases, nuclei — follows from *non-overlapping domains* in place of antisymmetry: "two electrons per orbital" becomes simply "two spatial territories," with no spin variable anywhere, and nothing missing. So the exclusion half of spin is *reducible to geometry*; it was doing bookkeeping that a spatial constraint does without it. This is why the whole of the programme has met no need for spin: everything built so far is the geometric half.

Magnetic-spin is irreducible. What is left over is the two-valued moment — observed directly and inescapably in Stern–Gerlach, in $g = 2$ to twelve digits, and in the everyday technology of EPR, NMR, and MRI. This is not bookkeeping; it is a physical degree of freedom, and it is precisely what RealQM does not yet carry. The residue is therefore *small and sharp*: not "spin" in its entirety, but only its SU(2) magnetic factor of two.

A conjecture on the connection — and a check that tempers it. If exclusion is geometric and the magnetic moment is a separate SU(2) structure, then the spin–statistics *connection* itself might seem an artefact of bundling the two under one name. It is tempting to conjecture so. But a test weighs against the strong form: closed-shell *diamagnetism* (§6) requires the two electrons' spins to pair antiparallel, and geometric non-overlap alone does not force this — so the connection reappears, as needed, precisely where more than one electron is magnetically active. We therefore state the position carefully: RealQM decouples exclusion from magnetism at the *single-electron* level (the caged moment vanishes, the free moment is a winding), but at the *many-electron* level the pairing that statistics enforces is still required for correct magnetism. We tested whether that pairing could be supplied *geometrically* (a shared-boundary coupling), and it cannot: a spin domain wall costs kinetic energy, so geometry favours parallel, not antiparallel (§6). The pairing

must be imported as a spin-statistics (exchange) term. The strong conjecture that the weld is mere convention is therefore *not* supported: geometric non-overlap carries the spatial consequences of antisymmetry but not its spin consequences, and closed-shell magnetism needs the latter.

3 Its experimental face: Stern–Gerlach

The residue is not academic; it is the whole content of the Stern–Gerlach experiment. A beam of silver atoms — each with a single valence electron in an $L = 0$ (orbitally spherical) ground state — passes through an inhomogeneous magnetic field and splits into *two* discrete spots. Two spots is the signature of a two-valued moment, $\mu_z = \pm\mu_B$, i.e. spin $\frac{1}{2}$ with $g = 2$; an orbital moment would split into an *odd* number $2l + 1$, and for the $L = 0$ silver electron would not split at all ($m = 0$, $\mu_z = 0$).

RealQM in its present form therefore *fails* Stern–Gerlach: with only orbital windings the $L = 0$ valence electron carries no moment and the beam would pass undeflected. Reproducing the two-spot pattern requires precisely the two-valued spin-$\frac{1}{2}$ structure that is missing. Stern–Gerlach and $g = 2$ are one problem: the two spots *are* $g = 2$ made visible.

Reading the spread: the forcing magnifies, the quantisation is magnified. The *separation* of the two spots, as opposed to their number, is ordinary magnetostatics. An atom with moment μ_z in a gradient $G = \partial_z B_z$ deflects by $z = \mu_z G L(D + \frac{1}{2}L)/Mv^2$ across a magnet of length L and drift D at speed v, so the split is

$$\Delta z = \frac{2\mu_B\, G\, L\, (D + \frac{1}{2}L)}{Mv^2}, \tag{1}$$

linear in the forcing G and $\propto 1/Mv^2$ (hence $\propto 1/T$ thermally, with the velocity spread giving each spot its width). The gradient thus sets the *scale* of the pattern, continuously; it does not set its *structure*. It never changes the count (two), never changes μ_z ($\pm\mu_B$), never fills the middle: stronger forcing only marches the same two dots farther apart, making the absent middle *more* conspicuous, not less. This is the clean factorisation the experiment performs — a continuous, classical magnifier (the deflection $\Delta z \propto G$, native to a real moment in a real gradient, $F = \mu_z G$) acting on a discrete, quantised input ($\pm\mu_B$). The spread is RealQM's for free; only the two *values* it magnifies are the residue.

4 What a charge density needs: two with no middle

It is worth pinning down exactly what a magnetic field asks of a charge density, because it isolates the residue to a single feature. A field couples to charge only through *motion* — through the current $\mathbf{J}$ — and three levels follow:

- **To react at all**, a density needs a current. A static real density ($\mathbf{J} = 0$) is magnetically inert to first order; a field exerts force and shifts energy only through $\mathbf{J}$ (the coupling $-\frac{q}{m}\mathbf{J}\cdot\mathbf{A}$, or the Lorentz $\mathbf{J}\times\mathbf{B}$). The one exception is diamagnetism — a weak *induced* current — which is uniform and does not split a beam.

- **To carry a moment**, the current must *circulate*: a phase winding $e^{im\phi}$ gives $\mu_z = -m\mu_B$. This a scalar RealQM density does whenever it has orbital circulation.
- **To split into two**, as in Stern–Gerlach, is the one thing a scalar circulation cannot do. A scalar winding admits $m = 0, \pm1, \pm2, \ldots$ — an *odd* ladder that *includes zero*. For $|m| \leq 1$ it gives three orientations ($-\mu_B$, 0, $+\mu_B$), three spots, with an undeflected *middle* beam. Stern–Gerlach gives two spots, $\pm\mu_B$, *with no middle*. That missing zero is the whole signature: it is the half-integer structure $m_s = \pm\frac{1}{2}$ (no $m_s = 0$), which a scalar density cannot produce.

So the residue reduces to a single phrase: **two with no middle**. Reacting to a field (a current) and carrying a moment (circulation) are ordinary moving-charge electromagnetism, already present in scalar RealQM; the irreducible extra is only the two-valued, zero-free structure of a half-integer spinor — the SU(2) feature, and nothing more. What Stern–Gerlach forbids is making that two-valuedness *vanish*; what it leaves open is whether the structure must be *posited* (as in the Dirac account) or can *emerge* from a two-valued, node-forced feature of a circulating configuration — the distinction between eliminating spin and *deriving* it (§6).

The angular momentum is already real (Einstein–de Haas). One might fear a deeper obstacle — that spin's *mechanical* angular momentum, the $\hbar/2$ made visible by the Einstein–de Haas effect (a freely suspended bar physically rotating as it is magnetised and its moments align), is a further ingredient the theory must supply beyond the moment itself. In RealQM it is not. A magnetic moment here *is* a real circulating charge, and a real current carries real mechanical angular momentum by construction — the same L_z that accompanies the orbital moment $\mu_z = -m\mu_B$. So the reciprocal exchange of angular momentum between the aligned moments and the body — Einstein–de Haas, and its converse the Barnett effect — is automatic, ordinary conservation for a charge in motion, requiring no spin primitive. This matters because it removes the last thing one might think spin has to be *posited* to provide: everything *mechanical* about spin is already real in RealQM. What remains genuinely owed is only the two features above — the two-valuedness with no middle, and the doubled magnitude $g = 2$ — both of which are the one SU(2) fact, not the reality of spin angular momentum. The residue is thus a single geometric *number*, not a missing substance.

5 The residue is geometric, not relativistic

It is tempting to conclude that spin, $g = 2$, and Stern–Gerlach force relativity into the theory, since in the textbook account $g = 2$ falls out of the Dirac equation. This conclusion is false, and the point is decisive for the present programme.

Lévy-Leblond. In 1967 Lévy-Leblond showed that $g = 2$ follows from a first-order (linearised) *non-relativistic*, Galilean spinor wave equation [4]. Relativity is sufficient to produce $g = 2$ but not necessary; the factor of two survives the non-relativistic limit because it does not come from Lorentz invariance in the first place.

The mechanism. Write the kinetic energy not as $p^2/2m$ but as $(\boldsymbol{\sigma}\cdot\mathbf{p})^2/2m$ — legal for free motion since $(\boldsymbol{\sigma}\cdot\mathbf{p})^2 = p^2$. In a magnetic field, minimal coupling $\mathbf{p} \to \mathbf{p} - q\mathbf{A}$ gives

$$\frac{(\boldsymbol{\sigma}\cdot(\mathbf{p}-q\mathbf{A}))^2}{2m} = \frac{(\mathbf{p}-q\mathbf{A})^2}{2m} \;-\; \frac{q}{2m}\,\boldsymbol{\sigma}\cdot\mathbf{B}, \tag{2}$$

and the last term is exactly the $g = 2$ spin–Zeeman coupling, appearing *automatically* the moment the dynamics is linearised in $\boldsymbol{\sigma}\cdot\mathbf{p}$. No relativity is used in (2); it is the ordinary Pauli Hamiltonian.

The geometry. The "2" is the **SU(2) double cover of the rotation group**: a spinor returns to itself only after 720°, rotating at half angle. That half is reciprocally the double in the moment. It is the same species of fact as the winding-number topology that fixes $\mu_z = -m\mu_B$ in the orbital case — a statement about how spin-$\frac{1}{2}$ objects transform, geometric and scale-free, owing nothing to rest energy.

6 The target for RealQM

The residue thus lands on the reachable side of the ledger. What RealQM needs is not relativity but the right *spinor structure* on its domains, and the base theory already points at it: the ψ_i on Ω_i acquiring an internal $\pm\frac{1}{2}$ label. The programme is to implement that label not as a bolt-on but as a $\boldsymbol{\sigma}\cdot\mathbf{p}$ linearisation of the one-electron dynamics on each domain, so that:

- the spin–Zeeman term $-\frac{q}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}$ of (2) appears, giving $g = 2$;
- the moment becomes *two-valued*, $\mu_z = \pm\mu_B$, so that an $L = 0$ electron in an inhomogeneous field splits into *two* beams — Stern–Gerlach — rather than none;
- the non-overlap/free-boundary structure, and the current $\mathbf{J}$ of the complex form, are retained unchanged, since (2) is a rewriting of the kinetic term, not a new interaction.

A first implementation (single electron). The single-electron step has been carried out numerically. On a two-dimensional grid a two-component spinor $\psi = (\psi_\uparrow, \psi_\downarrow)$ is evolved with the $\boldsymbol{\sigma}\cdot\mathbf{D}$ structure, $\mathbf{D} = \mathbf{p} - q\mathbf{A}$, in a uniform field. Two facts come out. First, the operator identity $(\boldsymbol{\sigma}\cdot\mathbf{D})^2 = \mathbf{D}^2 \not\vdash - q\,\boldsymbol{\sigma}\cdot\mathbf{B}$ holds to grid accuracy (residual $\sim 10^{-3}$, i.e. $O(dx^2)$), so the $g = 2$ Zeeman term $-q\,\boldsymbol{\sigma}\cdot\mathbf{B}$ *emerges* from squaring $\boldsymbol{\sigma}\cdot\mathbf{D}$ — it is not inserted by hand. Second, an $L = 0$ (orbitally inert) state — the very state a scalar density leaves undeflected — splits under the spinor into *two* levels $\pm\mu_B B$, splitting $2\mu_B B$, $g_s = 2$, with *no middle*, against the three-level ($m = -1, 0, +1$) orbital pattern. This is Stern–Gerlach: the $\boldsymbol{\sigma}\cdot\mathbf{p}$ structure delivers exactly the residue, non-relativistically. The distinguishing content is that spin reaches $\pm\mu_B$ at $\pm\frac{1}{2}$ unit of angular momentum ($g = 2$, no middle) where orbital needs ± 1 unit ($g = 1$, with middle).

We state plainly what remains a *target*. Two things are not yet settled. *(i) Many-electron consistency — and here the naive extension fails.* The spinor is a per-electron magnetic label, and it must coexist with the geometric realisation of Pauli exclusion by non-overlap *without double-counting the spin* — concretely, a filled shell of two electrons must pair to zero net moment (diamagnetism). We tested the *spin sector* of the naive version — two electrons, each carrying an independent spinor

with no coupling between them, in an external field, the spatial structure left aside — and it does *not* pass: the ground state is the aligned pair $\downarrow\downarrow$ with net moment $2\mu_B$, *paramagnetic*, where the observed closed shell is *diamagnetic*. (This is the spin bookkeeping alone; no concrete two-domain spatial system, such as the two Bernoulli half-domains of helium, was solved — which is exactly where a shared-boundary coupling might change the answer.) Nothing in geometric non-overlap forces the two spins antiparallel; obtaining diamagnetism requires an antiparallel-pairing (exchange) term $J\,\mathbf{S}_1{\cdot}\mathbf{S}_2$, which is a spin-statistics ingredient not supplied by non-overlap. Diamagnetism needs only the antiparallel *product* state $\uparrow\downarrow$ (net moment zero, not the entangled singlet). One might hope a *geometric* coupling — continuity of the spinor field across the shared boundary — would force the two domains antiparallel and supply the pairing without re-importing spin-statistics. We computed this, and it *fails*. Forcing the joined spinor field antiparallel means a spin *domain wall* across the boundary, whose kinetic cost is $T_{\text{spin}} = \frac{1}{8}\int(\theta')^2 \approx \pi^2/24w > 0$, while the parallel configuration costs nothing: continuity across the boundary therefore *favours parallel* (paramagnetic), the opposite of what is needed. And in the actual RealQM structure — two separate functions on non-overlapping domains — the spinors are simply uncoupled and degenerate, aligning with any field. Either way geometry does not deliver antiparallel pairing; if anything it leans the wrong way. The reason is structural and worth stating: geometric non-overlap reproduces the *spatial* consequences of antisymmetry (exclusion, shells, the whole of chemistry) but not its *spin* consequences (singlet pairing, exchange, closed-shell diamagnetism). Closed-shell magnetism therefore requires an *added* exchange term; it is a genuine limitation of the spin-blind geometry, not a gap awaiting a boundary condition. *(ii) Why the spinor.* Using $(\boldsymbol{\sigma}\cdot\mathbf{D})^2$ rather than $\mathbf{D}^2$ is here a choice; Lévy-Leblond *derives* it from Galilean covariance of a first-order equation, but whether RealQM's domain structure *forces* it or merely permits it is open. What is established is stronger than the map, however: the residue is not only geometric and non-relativistic but *concretely delivered* for one electron by a $\boldsymbol{\sigma}\cdot\mathbf{p}$ structure on the domains, with $g = 2$ emerging rather than posited. It belongs inside RealQM, and a working single-electron piece is now in hand.

7 Conclusion

The charges-as-configurations picture closes the free-charge and relativity problems and leaves a single, sharp residue: the electron's spin factor $g = 2$, whose experimental embodiment is the two-spot Stern–Gerlach splitting. That residue is not a relativistic quantity smuggled in through the Dirac equation; by Lévy-Leblond it is a non-relativistic, geometric consequence of the SU(2) spinor structure, delivered by a $\boldsymbol{\sigma}\cdot\mathbf{p}$ linearisation. RealQM therefore has a concrete, relativity-free target for spin: give its domains the $\pm\frac{1}{2}$ label as a $\boldsymbol{\sigma}\cdot\mathbf{p}$ structure, and $g = 2$ and Stern–Gerlach's two beams should follow together. Until that is carried through the multiphase problem the point is a target, not a result — but it is the right size of target, and it is inside the theory, not beyond it.

Put ontologically: spin need not be *posited* in RealQM at all. Its exclusion role is already geometry (non-overlapping domains); its mechanical angular momentum is already the real circulation of charge, so Einstein–de Haas and its converse are native rather than obstacles; and what is left is not a substance called "spin" but a single geometric *number* — the SU(2) factor $g = 2$, the two-valued moment with no middle — to be *derived* from the configuration, not imported. That

is the whole of what is owed.

References

[1] C. Johnson, *Complex Time-Dependent RealQM: Currents, Radiation, and Magnetism from the Real-Space Continuum*, 2026.

[2] C. Johnson, *Magnetism as Circulating Charge: Why the Nuclear Electron Carries No Magnetic Moment*, 2026.

[3] C. Johnson, *Elementary Charges as Configurations, not Objects: Why RealQM Needs No Relativity*, 2026.

[4] J.-M. Lévy-Leblond, *Nonrelativistic particles and wave equations*, Commun. Math. Phys. **6**, 286 (1967).