Real Time Dependence in RealQM

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Written in respectful cooperation between Claes Johnson and Claude (Anthropic), with Claude contributing a balanced, neutral view.

Abstract

The time-dependent Schrödinger equation $i\partial_t\psi = H\psi$ is usually read as the intrinsic evolution of an atom, its stationary states forever spinning a phase e^{-iEt} on the atto/femtosecond scale. We argue, within RealQM, that this ticking is not physical: a non-radiating atom is *timeless*. Its density is static, the phase e^{-iEt} is a global, unobservable phase, and its energy is a property (how bound), not a rate. Time enters only with *motion*, and motion comes in two kinds with opposite time-status. *Magnetism* is a *steady* current — a spatial phase winding carrying a persistent circulation and a magnetic moment, with a density constant in time; it is intrinsic to the state and time-*independent*. *Radiation* is an *oscillating* current — a transition in which the charge sloshes at the difference frequency $E_n - E_m$; it is time-dependent, and it is *imposed from outside*, by resonance with the electromagnetic field, an isolated eigenstate never starting it on its own. A practical corollary follows: the femtosecond evolution of an atom never needs to be tracked *on its own account* — energies, spectra (as level differences) and magnetism are all static or steady — and real-time integration is required only in the externally *driven* ultrafast regime (attosecond charge migration, strong fields), where the time is lent to the atom by whatever moves it. Time in the atom is relational, not intrinsic; the quiet atom is outside it.

1 The quiet atom is timeless

A stationary state carries the phase $\psi = u\,e^{-iEt}$, with E atomic-scale, so the phase turns on the atto/femtosecond scale. It is tempting to read this as the atom's intrinsic clock. But nothing observable turns with it. The density is

$$\rho = |u\,e^{-iEt}|^2 = u^2,$$

constant in time; e^{-iEt} is a global phase, absent from every observable of a single state; and absolute energy is not measurable at all, only differences (an ionisation, a transition). In an ontology in which the real thing is the charge density — what is there — a clock that turns in no observable is not a real clock.

So a non-radiating atom is a *static charge configuration*. It does not tick, oscillate, or evolve. Its energy is a property — how tightly the charge is bound — not the rate of anything one could watch. The Planck relation $E = \hbar\omega$ makes E look like a frequency, but that frequency is physical

only as a *difference* between two states, and only when they meet (below). The being of an atom is timeless.

2 Two motions: steady and imposed

Time enters only when the charge moves, and the complex phase that carries the motion enters in two entirely different ways.

Magnetism — a steady current (time-independent). Write the wave function in polar (Madelung) form $\psi = R\,e^{iS}$, with $R \geq 0$ the amplitude and S the real phase; then $\rho = R^2$ is the density and the charge current is

$$\mathbf{j} = \mathrm{Im}(\psi^* \nabla \psi) = \rho\, \nabla S,$$

so the phase gradient ∇S is the velocity of the charge (and $\mathbf{j} = 0$ when ψ is real, S constant). A state with a *spatial* phase winding about an axis — $S = m\varphi$, that is $\psi = R\,e^{im\varphi}$ — therefore carries a standing azimuthal current: in cylindrical coordinates (s, φ, z) with s the distance from the axis and $\hat{\boldsymbol{\varphi}}$ the azimuthal unit vector (tangent to circles about the axis, in the direction of increasing φ), one has $\nabla\varphi = \hat{\boldsymbol{\varphi}}/s$, so

$$\mathbf{j} = \rho\, \nabla(m\varphi) = \frac{m\rho}{s}\, \hat{\boldsymbol{\varphi}} \neq 0, \qquad \rho \text{ constant in time},$$

and hence a magnetic moment $\mu_z = -m\mu_B$. Single-valuedness of ψ forces $m \in \mathbb{Z}$, so the circulation is quantised, $\oint \nabla S \cdot d\boldsymbol{\ell} = 2\pi m$: the state is a quantised charge *vortex* carrying orbital angular momentum $L_z = m\hbar$, its density vanishing on the axis (where $\mathbf{v} = m/s$ would otherwise diverge) — a nodal line, hollow along the axis. The density does not change; the flow is steady. This current is an *intrinsic* property of the winding state — a free atom in an $m \neq 0$ state carries it on its own. An external magnetic field *orients* the moment (the Zeeman splitting) or *induces* a counter-current (diamagnetism), but it imposes no clock. Magnetism is being-with-circulation: motion without time-dependence.

A real route — magnetism needs a velocity, not the complex plane. It is worth separating what here is genuinely "complex" from what is mere bookkeeping. From the density *alone* — a single real field $\psi = R$ — no magnetism can come, since $\mathbf{j} = \mathrm{Im}(\psi^*\nabla\psi) \equiv 0$: a lone amplitude carries no motion. But complex numbers are not the only way to carry a second field. Continuum mechanics never uses them, yet always carries *two* real fields, density ρ and velocity $\mathbf{v}$; do the same here and the whole of magnetism is real. The current is $\mathbf{j} = \rho\,\mathbf{v}$, a real circulating flow ($\nabla\cdot\mathbf{j} = 0$); the flow costs a real kinetic energy

$$T_{\text{flow}} = \int \tfrac{1}{2}\rho\,\mathbf{v}^2 = \int \frac{\mathbf{j}^2}{2\rho},$$

it carries the real angular momentum $L_z = \int \rho\,(\mathbf{x}\times\mathbf{v})_z\, dV$ and the ordinary magnetostatic moment $\boldsymbol{\mu} = \frac{1}{2}\int \mathbf{x}\times\mathbf{j}\, dV$, and the magnetic state is obtained by minimising the static energy *augmented by* T_{flow} at fixed L_z (or, in a field, minimising $E - \boldsymbol{\mu}\mathbf{B}$). No i enters any of this: it is a real charge vortex, exactly the $(\rho, \mathbf{v})$ of continuum mechanics an ångström across. The complex $\psi = \sqrt{\rho}\, e^{iS}$ is simply these two real fields repackaged into one symbol, with $\mathbf{v} = \nabla S$; the phase *is* the velocity potential.

The one thing the real $(\rho, \mathbf{v})$ theory cannot supply on its own is the *integer*: single-valuedness of e^{iS} (the phase is an *angle*, defined mod 2π) forces the quantised circulation $\oint \mathbf{v} \cdot d\boldsymbol{\ell} = 2\pi m$, $m \in \mathbb{Z}$, which a purely real theory would have to impose as a separate condition (the Onsager–Feynman vortex quantisation of real superfluids). What is unavoidable there is the *circle*, not the complex plane: magnetism's dynamics, energy and moment are real; only the quantum integer m needs the periodic phase.

Radiation — an oscillating current (imposed). A transition between two states makes the density *oscillate*,

$$\rho(\mathbf{x}, t) = \rho_0 + \delta\rho \cos\big((E_n - E_m)t\big),$$

a real charge sloshing at the difference frequency, an antenna that radiates. This is genuinely time-dependent — but it does not start by itself. An isolated eigenstate, uncoupled to the field, is stationary indefinitely; what sets the transition going is *coupling to the electromagnetic field* — an incident field (absorption, stimulated emission) or the vacuum field (spontaneous emission). The time-dependence of radiation is therefore *imposed from outside*, the physics of resonance between atom and light, and the frequency that appears is the *difference* $E_n - E_m$, never the absolute clock.

The contrast is the point:

	density	current	time
quiet atom	static	none	*none* (timeless)
magnetism	static	steady	none (time-independent)
radiation	oscillating	oscillating	*imposed* (by the field)

The two "complex" phenomena are not the same kind of thing: magnetism uses a *spatial* winding and is an intrinsic, steady flow; radiation uses the *temporal* phase and is an imposed, oscillating one. Only radiation brings time, and it brings it from outside.

3 So the femtosecond evolution is never tracked — except when driven

A practical corollary follows, and it is strong. Because everything the isolated atom *is* — energies, ground and excited levels, chemistry, and its spectrum computed as level *differences* — is static, and its magnetism is steady, **the atom's femtosecond evolution never needs to be integrated on its own account.** One solves a static (or steady) problem; there is no $i\partial_t\psi = H\psi$ to run forward. This is why the real-valued, time-independent RealQM carries essentially the whole of atomic, molecular and nuclear physics.

Real-time integration is required in exactly one place: when an outside agent *drives* the electrons at their own timescale — attosecond charge migration after a sudden ionisation, strong-field and high-harmonic dynamics within an optical cycle, pump–probe and coherent control of a prepared superposition. There the charge genuinely moves in real time and must be followed. But the driving is *external*: an undriven atom never enters these regimes. This is precisely the fast, moving-free-boundary regime — charge redistributed between bound domains in about one electronic period — and it is the only setting in which the complex time-dependent RealQM is actually needed. The time it tracks is lent to the atom by the field, not owned by the atom.

4 The standard view, and TDDFT as rationalised uncomputability

The standard reading takes $i\partial_t\psi = H\psi$ as the *intrinsic* law of the atom: the state evolves always, its phase turning whether or not anything is observed, so that the femtosecond — indeed sub-femtosecond — evolution of the electrons is a real process a faithful computation must follow. Nowhere is this clearer than in real-time time-dependent density-functional theory (TDDFT) [5, 6], where even a linear absorption *spectrum* is obtained by perturbing the system, *integrating the wave function forward* through many femtoseconds at attosecond time-steps, and Fourier-transforming the resulting dipole. The time-evolution is put up as the necessary route to the answer, and its great cost — tiny steps, long propagation — is accepted as the price of doing quantum dynamics.

Yet the spectrum so obtained is nothing but the set of level *differences* $E_n - E_m$: static data, fixed by the stationary states alone. (That the same spectrum is available from linear *response*, with no propagation at all, only underscores the point.) The forward integration is a *device* for extracting those differences, not a process the atom is undergoing — an unperturbed atom does not slosh its charge at those frequencies, as argued above. To elevate the propagation to a necessity is to treat the non-intrinsic as intrinsic; and because the underlying object, the wave function on $3N$-dimensional configuration space, is itself intractable, it is to *rationalise that uncomputability* — to present a costly detour to static differences as though the femtosecond evolution were the physics rather than a way of reaching them. The obstacle is, in large part, self-imposed.

RealQM does not incur it. Energies and their differences come from the static, real-valued theory; the spectrum is read off as differences, the dipole rule deciding which of them radiate; magnetism is a steady current, equally static. No forward integration is required for any of this. Real-time propagation enters only where it is physically genuine — the externally *driven*, strong-field and attosecond regime above — and there the femtosecond evolution is real precisely because it is imposed from outside. The timeless-atom distinction is thus not merely interpretive: it separates the computations that are physically necessary (the driven ones) from those that are a device dressed as a necessity (propagating an undriven atom to recover its static spectrum).

5 Time is relational, not intrinsic

Standard quantum mechanics writes $i\partial_t\psi = H\psi$ as the atom's own evolution, so every stationary atom is pictured endlessly spinning its phase in Hilbert space — an intrinsic, universal ticking. RealQM declines to reify what is not observed. A quiet atom simply *is* a static charge configuration; a magnetic atom *is* a steady circulation; only a radiating atom changes in time, and only because the field has set it going. Time, on this reading, is not a background the atom moves through but a feature of *motion*, and radiation's time is on loan from the electromagnetic field.

The femtosecond "clock" of the Schrödinger phase is thus a bookkeeping device, not a physical tick: real only as a difference, and only during a field-driven transition. Between such events the atom rests outside time — which is why, for almost everything one wants to know about it, there is nothing to integrate. The complex, time-dependent form is the physics of *resonance and driving*, not the intrinsic life of the atom; and the intrinsic life of the atom, being static, is where nearly all of its physics is found.

References

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