Magnetism without the Complex Plane

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Abstract

Orbital magnetism is routinely presented as intrinsically complex: a circulating current needs a phase, and a phase means a complex wave function $\psi = \sqrt{\rho}\, e^{iS}$. We separate, within RealQM, what is genuinely quantum here from what is mere bookkeeping. From the charge density *alone* — a single real field — no magnetism can come, since the current $\mathbf{j} = \mathrm{Im}(\psi^* \nabla \psi)$ vanishes identically for real ψ: a lone amplitude carries no motion. But complex numbers are not the only way to carry motion. Continuum mechanics never uses them, yet always carries *two* real fields, a density ρ and a velocity $\mathbf{v}$; carried the same way here, the whole of orbital magnetism is real. The current is $\mathbf{j} = \rho\, \mathbf{v}$, a real circulating flow; it costs a real kinetic energy $\int \frac{1}{2}\rho|\mathbf{v}|^2$; it carries a real angular momentum and the ordinary magnetostatic moment; and the magnetic state is obtained by minimising the static energy augmented by that flow energy at fixed angular momentum. No i enters any of it: magnetism is a real charge vortex, the $(\rho, \mathbf{v})$ of continuum mechanics an ångström across. The complex $\psi = \sqrt{\rho}\, e^{iS}$ is exactly these two real fields repackaged into one symbol, the phase being the velocity potential. The single thing the real theory cannot supply on its own is the *integer*: single-valuedness of e^{iS} forces the quantised circulation $\oint \mathbf{v} \cdot d\boldsymbol{\ell} = 2\pi m$, $m \in \mathbb{Z}$, which a purely real formulation must impose as a separate condition (the Onsager–Feynman vortex quantisation of real superfluids). What is unavoidable there is the *circle*, the periodicity of an angle, not the complex plane. Magnetism's dynamics, energy and moment are real; only the quantum integer needs the periodic phase.

1 The question

Magnetism is charge in motion, and motion in quantum mechanics is carried by a phase: the standard route to an orbital magnetic moment runs through a complex, phase-winding wave function $\psi = R(\mathbf{x})\, e^{im\varphi}$, whose current and moment follow from the imaginary part of $\psi^* \nabla \psi$. It is natural to conclude that the complex wave function is *necessary* for magnetism — that here, at least, RealQM must leave the real, static picture behind and become complex.

The conclusion is half right and worth taking apart, because the halves belong to different things. Something beyond the bare density is indeed needed. But that something is a *velocity*, not a complex number; and complex arithmetic, where it appears, is a convenient packaging of two real fields, not an irreducible ingredient. We make the separation explicit: what magnetism needs, what the complex form merely bundles, and the one genuinely quantum residue that the real theory cannot generate on its own.

2 From the density alone: nothing

Let the only field be the real amplitude, $\psi = R$, so that the charge density is $\rho = qR^2$. The quantum-mechanical current is

$$\mathbf{j} = \frac{q}{m_e}\,\mathrm{Im}(\psi^*\nabla\psi),$$

and for real ψ this is *identically zero.* A single real field has no current in it and therefore no magnetic moment. This is the true content of the slogan "magnetism needs the complex form": from the density by itself, nothing circulates, because a lone amplitude encodes no motion. Whatever carries magnetism must be a *second* field, over and above ρ.

3 Two real fields: $(\rho, \mathbf{v})$ continuum mechanics

Complex numbers are one way to carry a second field, but not the only way. Macroscopic continuum mechanics carries motion with no complex numbers at all: it works throughout with a density ρ and a velocity $\mathbf{v}$. Carry the same pair in RealQM and the whole of orbital magnetism is real. The current is a real circulating flow,

$$\mathbf{j} = \rho\,\mathbf{v}, \qquad \nabla\cdot\mathbf{j} = 0 \text{ (stationary)}, \tag{1}$$

a charge vortex. The flow costs a real kinetic energy,

$$T_{\text{flow}} = \int \tfrac{1}{2}\,\rho\,|\mathbf{v}|^2\,d^3x = \int \frac{|\mathbf{j}|^2}{2\rho}\,d^3x, \tag{2}$$

it carries a real angular momentum and the ordinary magnetostatic moment of a current distribution,

$$L_z = \int \rho\,(\mathbf{x}\times\mathbf{v})_z\,d^3x, \qquad \boldsymbol{\mu} = \tfrac{1}{2}\int \mathbf{x}\times\mathbf{j}\,d^3x, \tag{3}$$

and the magnetic state is the stationary point of the *static* RealQM energy augmented by the flow energy at fixed angular momentum,

$$\min\,\big\{E_{\text{static}}[\rho] + T_{\text{flow}}[\rho,\mathbf{v}] \;:\; L_z \text{ fixed}\big\}, \qquad \text{or, in a field,}\quad \min\,\big(E - \boldsymbol{\mu}\cdot\mathbf{B}\big). \tag{4}$$

No i enters (1)–(4). This is a real charge vortex, precisely the $(\rho, \mathbf{v})$ of continuum mechanics scaled down to atomic size — the same object a fluid dynamicist would write for a swirling incompressible flow, now made of charge and confined to an atom.

4 What the complex form packages

Where, then, does the familiar complex wave function come in? Write it in polar (Madelung) form,

$$\psi = \sqrt{\rho}\,e^{iS}, \qquad \mathbf{v} = \frac{\nabla S}{m_e},$$

and the identification is immediate: the modulus is the (root) density, and the phase S is nothing but the *velocity potential*, its gradient the velocity. The current $\mathbf{j} = \rho\,\mathbf{v}$ of (1) is exactly $\mathrm{Im}(\psi^*\nabla\psi)$

times the charge-to-mass factor. So $\psi = \sqrt{\rho}\, e^{iS}$ is not a new ingredient at all: it is the two real fields $(\rho, \mathbf{v})$ of §3 bundled into a single complex symbol. The "i" is bookkeeping — a compact way to carry an amplitude and a flow together — and the physics is the flow. (This is also why, numerically, the honest integrator uses the Cartesian real pair $(u, v) = (\operatorname{Re}\psi, \operatorname{Im}\psi)$, smooth through nodes, rather than the polar form, which is singular exactly on the vortex axis where the amplitude vanishes.)

5 The one quantum residue: the integer

There is a single thing the real $(\rho, \mathbf{v})$ theory cannot supply on its own, and it is worth naming precisely. A winding state has phase $S = m\varphi$ about an axis, giving in cylindrical coordinates (s, φ, z) the azimuthal velocity $\mathbf{v} = (m/m_e s)\,\hat{\boldsymbol{\varphi}}$ and hence $\mu_z = -m\mu_B$, quantised in m. Why must m be an *integer*? Because S is an *angle*: the phase is defined only modulo 2π, equivalently e^{iS} must be single-valued, and this forces the circulation to be quantised,

$$\oint \mathbf{v} \cdot d\boldsymbol{\ell} = \frac{2\pi m}{m_e}, \qquad m \in \mathbb{Z}, \tag{5}$$

and with it $L_z = m\hbar$ and the moment in whole units of μ_B. A purely real formulation carrying only $(\rho, \mathbf{v})$ would have every ingredient of the vortex except this: it would have to *impose* (5) as a separate quantisation condition — which is exactly what one does for the quantised vortices of a real superfluid (Onsager and Feynman). And what is genuinely required there is the *circle*, the periodicity of an angle, not the complex plane as such. The density must also vanish on the axis (a nodal line, the hollow vortex core), since $\mathbf{v} \sim 1/s$ would otherwise diverge; that too is a consequence of the winding, not of complex arithmetic.

6 Consequences

The upshot sharpens RealQM's guiding slogan — the same 3D form as macroscopic continuum mechanics, only smaller. Continuum mechanics is $(\rho, \mathbf{v})$; a steady magnetic current is therefore *native* to the real theory, not an import from a complex one. Orbital magnetism — moments, angular momentum, the Zeeman response to an applied field, diamagnetic counter-currents — is real continuum mechanics; the sole quantum input is the integer winding number, and that input is the periodicity of an angle.

Two boundaries keep the account honest. First, this covers the *steady* member of the "complex" family only. *Radiation* is the *oscillating* member: a transition makes the density itself oscillate in time at $E_n - E_m$, and a genuinely time-varying density does require the complex, time-dependent form, since a real currentless density is static and dark [4]. The temporal phase there is real physics (imposed by the field), whereas the spatial phase of magnetism is, as shown, repackaged real flow. Second, single-electron *spin* ($g = 2$, Stern–Gerlach) is a further residue, reached from a spinor structure and not from the scalar $(\rho, \mathbf{v})$ picture of orbital magnetism [2]. Within those boundaries the statement stands: magnetism's dynamics, energy and moment are real; only the quantum integer needs the periodic phase.

7 A numerical test: diamagnetic susceptibility

The account can be held to a real number, read straight from the ground-state density with no field applied. The Langevin–Larmor molar diamagnetic susceptibility is

$$\chi_M = -\frac{N_A e^2}{6 m_e c^2} \sum_i \langle r_i^2 \rangle = -0.792 \times 10^{-6} \Big(\textstyle\sum_i \langle r_i^2 \rangle / a_0^2 \Big) \ \mathrm{cm}^3/\mathrm{mol}, \tag{6}$$

a sum over electrons of the mean-square radius — precisely the real-space quantity RealQM supplies as $\langle r^2 \rangle = \int r^2 \rho \, d^3x$. We evaluate $\sum_i \langle r_i^2 \rangle$ for the closed-shell atoms He, Ne, Ar from a radial RealQM self-consistent field in the spirit of §3–§4: each electron moves in the kernel $-Z/r$ plus the Hartree field of *all the other* electrons, with no self-repulsion (the RealQM rule) and no exchange. The result feeds (6) directly:

atom	$\sum \langle r^2 \rangle$ (a_0^2)	χ_{calc}	χ_{exp}	ratio
He	2.38	-1.88	-1.90	0.99
Ne	11.14	-8.82	-7.20	1.23
Ar	31.66	-25.1	-19.6	1.28

χ in 10^{-6} $\mathrm{cm}^3/\mathrm{mol}$; experiment from tabulated molar susceptibilities.

Reading the result honestly. For **helium the prediction is essentially exact** ($0.99\times$ experiment): the two-electron no-self-interaction field gives $\langle r^2 \rangle = 1.19 \, a_0^2$ per $1s$ electron, in agreement with Hartree–Fock and with the measured susceptibility. This is a magnetic observable *computed* from a RealQM density and matched to measurement, not asserted. For **neon and argon the size and trend are right but the atoms come out $\sim$20–30% too diamagnetic** ($1.23\times$, $1.28\times$); the per-shell breakdown pins the cause on the *valence* ($2p$ for Ne, $3p$ for Ar) being too diffuse. Two caveats frame this. First, the computation is a *radial mean field*: it enforces the no-self-interaction rule but spherically averages over the non-overlap (domain) geometry and omits exchange, both of which would *contract* the valence — so this first pass is closer to a lower bound on the full 3D multiphase solver's accuracy than to its verdict. Second, the trend — excellent for the two-electron atom, degrading for the many-electron ones absent the full geometry — is the same regime boundary RealQM shows elsewhere. The test is therefore a genuine partial success: one magnetic number secured against experiment (He), the right order and trend for Ne and Ar, and the missing physics (valence contraction) named rather than hidden.

8 What is new, what is not, and the boundary

Because the phenomenology here is standard, it is worth stating plainly what RealQM does and does not add, so the contribution is not mistaken either way.

What is genuinely new is the ontology. In the usual account the orbital moment is an operator $\hat{\mathbf{L}}$ on an amplitude, and the "current" is a *probability* current — physical for one particle, and otherwise a construct on $3N$-dimensional configuration space. RealQM makes magnetism a literal *charge fluid* $(\rho, \mathbf{v})$ circulating in ordinary three-dimensional space, one genuine current per electron for any N: a real charge vortex, the same object continuum mechanics writes for a swirling flow, an ångström across [1]. From this follow two clarifications with no counterpart in the standard telling. First, the complex wave function is, for *steady* magnetism, mere bookkeeping: what is needed is a velocity, not an i, and $\psi = \sqrt{\rho}\, e^{iS}$ merely bundles $(\rho, \mathbf{v})$, so the only irreducibly quantum residue is the integer m — forced by the *circle* (the periodicity of an angle), not by the complex plane. Second, this locates magnetism precisely against radiation: both belong to the "complex family," but magnetism is a *spatial* winding (steady, real flow in disguise) whereas radiation is a *temporal* oscillation that genuinely requires the complex time-dependent form, a real currentless density being dark [4]. Same family, opposite time-status.

What is not new is the phenomenology. RealQM *reproduces*, it does not rewrite, the standard results. For a reader meeting these by name, they are, in plain terms:

- *The gyromagnetic relation* $\boldsymbol{\mu} = (q/2m_e)\mathbf{L}$: a circulating charge is a tiny current loop, and a current loop is a small magnet, so the magnetic moment $\boldsymbol{\mu}$ is proportional to the orbital angular momentum $\mathbf{L}$ (the amount of circulation), with a fixed constant $q/2m_e$. More spin-around means a stronger magnet.

- *Quantised circulation*: the winding cannot be any amount but only whole multiples $m = 0, \pm 1, \pm 2, \ldots$ (shown above), so the moment comes in discrete steps $\mu_z = -m\mu_B$ rather than a continuum — the atomic-scale graininess seen, e.g., in the Stern–Gerlach experiment.

- *Zeeman splitting*: placed in a magnetic field B, states that differ only in m acquire slightly different energies $(-m\mu_B B)$, so a single spectral line splits into several. Measuring the split is how field strengths are read off spectra — for instance the magnetic fields of sunspots.

- *Langevin diamagnetism* $\chi \propto -\langle r^2 \rangle$: even an atom with no net moment develops a weak moment *opposing* an applied field (an induced counter-current), so it is faintly repelled by a magnet; the strength of this response scales with the atom's mean-square size $\langle r^2 \rangle$. This is why noble gases and most everyday molecules are weakly diamagnetic.

The hydrodynamic $(\rho, \mathbf{v})$ form is Madelung's [6]; quantised vortices are Onsager's and Feynman's [7, 8]. No new magnetic number or law is claimed here; the novelty is in *what these quantities are*, not in their values.

The boundary is collective magnetism. What RealQM does *not* supply is exchange-driven magnetism — the singlet–triplet splitting, ferromagnetic ordering — which requires spin exchange that geometric non-overlap does not deliver. Single-electron spin is reachable as a further residue ($g = 2$, without relativity), but the collective effects are not, and this note claims no perspective that resolves them [2]. The account of orbital magnetism above is complete and real; the exchange sector remains open, and is named as such.

References

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