Muon-Catalyzed Fusion as a Change of Glue: the Deuteron Molecular Ion Rescaled

Claes Johnson[1]

[1]KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden ,
claesjohnson@gmail.com

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Physical argument and model by the author; numerical implementation with AI assistance (Claude, Anthropic).

Abstract

In the RealQM/RealNucleus picture a deuteron molecular ion — two deuterons bound by a single shared negative charge — is a *glued pair*, and the glue's kinetic–Coulomb balance fixes the internuclear separation R_e. The glue enters through a single *scale* c, the coefficient of its kinetic energy, and the molecule inherits it: $R_e \propto c$, a smaller-scale glue making a smaller molecule. The electron and the muon are both unit negative charges differing only in this scale — the muon's is empirically about 200× smaller (muonic atoms are ∼200× smaller than electronic ones) — so the muon-bound ion is the electron-bound ion contracted by that factor: from $R_e \approx 2\,a_0 \approx 1.06$ Å for dd$\,e$ (the D_2^+ analogue) to $R_e \approx 512$ fm for ddμ, a ∼200× linear compression, $\sim 10^7$ in volume. At that separation the residual Coulomb barrier between the two deuterons is thin enough to be tunnelled fast compared with the muon lifetime, and the pair fuses: this is *muon-catalyzed fusion*. We compute the two molecular ions in RealQM as one Coulomb functional evaluated at two glue scales, and connect the catalytic cycle to its intrinsic limits — the muon lifetime and α-sticking — which cap it below break-even. In keeping with the unified Coulomb picture, the operative variable throughout is the glue's *size*, not a mass in the Coulomb law: the muon is simply a smaller electron.

Keywords: muon-catalyzed fusion; deuteron molecular ion; RealQM; RealNucleus; Coulomb binding; glue size; tunnelling; scale c.

1 Introduction

Ordinary hydrogen molecular ions do not fuse: in H_2^+ or D_2^+ the two nuclei sit about an ångström apart, held there by a shared electron, and the Coulomb barrier between them is far too thick to tunnel on any relevant timescale. Replace that electron with a muon — a lepton identical in charge but living at a scale some 200 times smaller (muonic atoms are ∼200× smaller than electronic ones) — and the same molecule shrinks by that factor, the two nuclei are drawn to within a few hundred femtometres, and they fuse essentially at once. A single muon, released when the pair fuses, goes on to catalyse further fusions until it decays. This is *muon-catalyzed fusion* (μCF), predicted by Frank [1] and Sakharov [2] and observed by Alvarez [3], with the modern theory established by Jackson [4] and the deuterium–tritium cycle measured and reviewed extensively [5, 6].

The purpose of this paper is to show that μCF has an especially transparent reading in the RealQM/RealNucleus framework [9, 7], where nuclei and their binding are charge clouds interacting by the Coulomb force alone. There a molecular ion is a *glued pair*: two positive nuclei confined by one negative charge cloud, the charge-conjugate of the way an atom confines its electrons. The negative glue is not a spectator but the bond, and its *size* — the single scale c of its kinetic energy — sets the size of the bond. Muon catalysis is then simply this molecule evaluated with a smaller-scale glue: one number, c, contracts the electron-bound ångström molecule into the muon-bound femtometre molecule that fuses. This is the same size $\propto c$ law by which, in the unified Coulomb picture [8], the atom and the nucleus differ; conventionally one writes $c = \hbar^2/2m$ and so labels the glue's scale by a mass, but that mass belongs, with gravitation, to a sector separate from electromagnetism, and the operative variable here is the size, not a mass in the Coulomb law. We make this quantitative, exhibit the size law and the accompanying energy law, and give the catalytic cycle its due together with the limits that keep it below break-even.

2 The molecular ion as a glued pair

In RealQM the ground state of a system of charges is a set of one-particle charge clouds ψ_i on non-overlapping domains with free boundaries, minimising the Coulomb energy

$$E \;=\; \sum_i c_i \int |\nabla \psi_i|^2 \;+\; \frac{1}{2} \iint \frac{\rho(x)\rho(y)}{|x-y|}\, dx\, dy, \qquad \rho = \sum_i q_i \psi_i^2, \qquad c_i = \frac{\hbar^2}{2m_i}, \tag{1}$$

with $q_i = \pm 1$ [9, 7]. The deuteron molecular ion is the charge-conjugate of a two-electron atom: two positive deuterons (each of unit charge, treated as fixed centres at separation R in the Born–Oppenheimer sense) and one negative glue cloud that pools between them. Minimising (1) over the glue at fixed R gives the molecular potential curve $E(R)$, whose minimum defines the equilibrium separation R_e: the balance between the glue's kinetic pressure, which resists being squeezed into the internuclear region, and the Coulomb attraction that draws it there. For the electron-bound ion this is the textbook D_2^+ curve, with

$$R_e^{(e)} \;\approx\; 2.0\, a_0 \;\approx\; 1.06 \text{ Å} \;=\; 1.06 \times 10^5 \text{ fm}, \tag{2}$$

a well of depth a few electron-volts. At this separation the two deuterons are 10^5 fm apart; the Coulomb barrier between them is enormous and fusion is utterly negligible. The molecular ion is stable, and that is all.

3 Changing the glue: the size law

The single feature of (1) that matters here is that the glue enters only through its scale c, the coefficient of its kinetic energy. Under a rescaling of length $R \mapsto \lambda R$ of a normalised cloud, the kinetic term scales as λ^{-2} and the Coulomb term as λ^{-1}, so the balance that fixes the molecule obeys

$$R_e \;\propto\; c, \qquad E_{\min} \;\propto\; \frac{1}{c}. \tag{3}$$

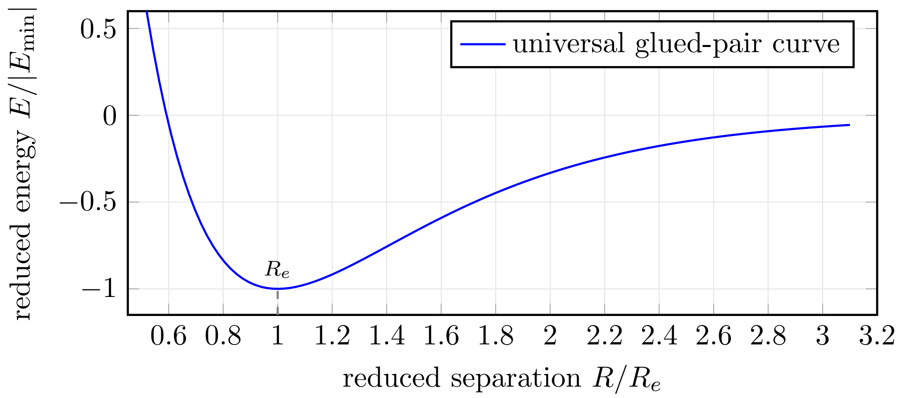

Figure 1: The molecular potential of a glued pair in reduced variables is universal: one curve for any glue. Only the scales move — the equilibrium separation $R_e \propto c$ and the well depth $|E_{\min}| \propto 1/c$ with c the glue's scale. For the electron $R_e \approx 1.06$ Å; for the muon, a $\sim 200\times$ smaller negative charge, the *same* curve sits at $R_e \approx 512$ fm and is $\sim 200\times$ deeper.

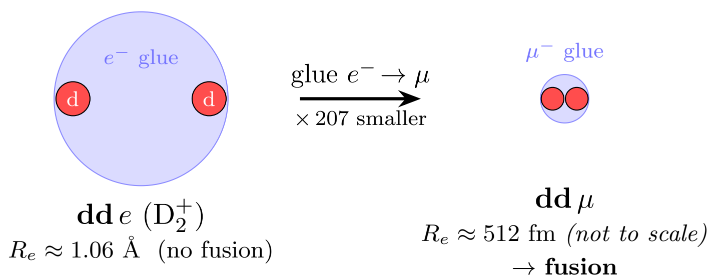

Figure 2: Muon catalysis as a change of glue. The electron-bound deuteron molecular ion holds its two deuterons about an ångström apart and does not fuse; replacing the electron with a muon — a $\sim 200\times$ smaller negative charge — contracts the same Coulomb molecule to ~ 512 fm (drawn far larger than to scale), close enough for the residual barrier to be tunnelled and the pair to fuse.

The molecular potential curve is otherwise *universal*: plotted in reduced variables R/R_e and $E/|E_{\min}|$ it is one and the same curve for any glue (Fig. 1); only the two scales R_e and $|E_{\min}|$ move, directly and inversely with the glue scale c. This is how the muon case is obtained in RealQM: the electron-bound $E(R)$ computed on the grid is re-read at the muon's scale $c_\mu \approx c_e/200$ — the empirical scale ratio of muonic to electronic binding — which contracts R_e and deepens the well by that factor. The result is

$$R_e^{(\mu)} = \frac{R_e^{(e)}}{200} \approx \frac{1.06 \text{ Å}}{206.8} \approx 512 \text{ fm}, \tag{4}$$

a linear compression of ~ 200 and a volume compression of $\sim 10^7$ (Table 1, Fig. 2). The muon-bound ddμ molecule holds its two deuterons a few hundred femtometres apart — four orders of magnitude closer than the electron holds them — purely because the glue is smaller. This is the same statement as the atom-versus-nucleus scaling of the unified Coulomb picture [8]: size $\propto c$, and the muon molecule is the electron molecule read at a smaller glue scale c.

glue	scale (rel.)	R_e	lifetime	fate
electron e^-	1	$2\,a_0 \approx 1.06 \times 10^5$ fm	stable	long-lived but too big: no fusion
muon μ^-	1/207	≈ 512 fm	2.2 μs	small *and* long-lived enough: fuses
tau τ^-	1/3477	≈ 30 fm	0.29 ps	small enough but decays first: no catalysis

Table 1: The deuteron molecular ion with the three charged-lepton glues. Length scales as the glue scale c (depth as $1/c$), per Eq. (3); only the glue's size differs. The muon is the sweet spot — the only lepton at once small-scale enough to compress the pair and long-lived enough to act; the electron is too big and the tau, though smaller still, decays $\sim 10^7$ times too fast to be captured. (Conventionally the scale is written $c = \hbar^2/2m$, so the muon's smaller scale is labelled by its larger mass.)

4 From separation to fusion

Compression buys fusion because the tunnelling probability through the residual internuclear Coulomb barrier is exponentially sensitive to the separation at which the two deuterons are held. Schematically the barrier-penetration factor is $\exp(-G)$ with a Gamow exponent $G \propto \sqrt{R_e}$ for a Coulomb barrier down to nuclear contact, so contracting R_e by ~ 200 raises the penetrability by a huge factor and turns an immeasurably slow process into a fast one. Quantitatively the muon-bound ddμ molecule fuses at a rate of order 10^9 s^{-1}, and the analogous dtμ (deuteron–triton) molecule — the practical choice, and the one on which reactor-scale μCF studies concentrate — at $\sim 10^{12}$ s^{-1} [6]. Either rate is enormous compared with the inverse muon lifetime, $1/\tau_\mu \approx 4.5 \times 10^5$ s^{-1} ($\tau_\mu = 2.2\ \mu$s), so once the muon-bound molecule forms, fusion is essentially certain. The electron-bound molecule, by contrast, has R_e larger by ~ 200 and a penetrability smaller by astronomical factors: it never fuses. The whole difference is the glue's scale acting through Eq. (3).

The internal structure of each fusing deuteron, and the fusion itself, are treated in RealNucleus in the same Coulomb terms: a deuteron is $2p + 1e$, and $\mathrm{d} + \mathrm{d} \to {}^4\mathrm{He}$ is computed directly as the merging of two such dual-confined units into the alpha [7]. The muon in μCF plays no part in that final step; it is the *delivery* mechanism — the heavy glue that brings the two deuterons into contact — and it is released when they fuse.

5 The catalytic cycle and its limits

The muon is a catalyst in the exact chemical sense. Liberated when the pair fuses, it is free to bind another two nuclei into a new tiny molecule and drive another fusion, and so on, once per cycle of duration set by the molecular-formation and fusion rates. Two facts bound how many times this can happen, and they are the reason μCF, though real and demonstrated, is not a power source.

The muon decays. With a lifetime $\tau_\mu = 2.2\ \mu$s the muon can complete only a finite number of cycles; at the achievable cycling rates this is of order 10^2–10^3 fusions per muon before the muon is gone.

The muon sticks. With small but nonzero probability the muon, instead of being released, remains bound to the newly formed α (a muonic helium ion) and is lost to catalysis. For the deuteron–triton cycle this α-sticking probability is $\omega_s \approx 0.4$–0.6%, which caps the number of catalysed fusions at roughly $1/\omega_s \approx 150$–250 regardless of the muon lifetime.

At $\sim$150 fusions of 17.6 MeV each, one muon yields $\sim$2.6 GeV, while producing a muon costs of order several GeV once accelerator and pion-production inefficiencies are included. The cycle therefore lands near, but below, break-even. Nothing in the RealNucleus reading changes this accounting: the framework clarifies the *mechanism* — fusion as a molecule with smaller glue — but the Coulomb barrier, the muon lifetime, and the sticking loss are all still there. There is no free lunch, only a clear picture of the trade.

6 What the RealNucleus reading adds

The value of the picture is conceptual economy. Muon catalysis is often presented as a special low-energy nuclear phenomenon; here it is nothing but the deuteron molecular ion of Eq. (1) with one number changed, the glue scale c. The same size $\propto c$ law that shrinks the atom to the nucleus in the unified Coulomb picture [8] shrinks the electron molecule to the muon molecule, and the muon molecule fuses for the same reason a nucleus is small: it lives at a smaller scale. Two consequences follow that are worth developing. First, the framework makes the *glue* the controllable variable, and the three charged leptons span a clean *size–lifetime trade-off* (Table 1). Smaller scale means tighter compression: the tau, at $\sim 1/3477$ the electron scale, would hold the pair at $\sim$30 fm, tighter even than the muon's $\sim$512 fm. But smaller-scale leptons are shorter-lived — the tau survives only $\sim$0.29 ps, some 10^7 times less than the muon, far too briefly to be slowed, captured, and formed into a molecule before it decays. The electron, at the other end, is stable but too large to bring the deuterons near. The *muon alone* is both small enough to compress the pair and long-lived enough (2.2 μs) to act — which is exactly why it, and not its lighter or heavier siblings, is the practical catalyst. Second, the framework places ordinary *electron screening* enhancement of fusion — a measured effect in which bound or plasma electrons modestly lower the barrier — on the same axis as muon catalysis: both are the negative glue drawing the nuclei together, differing only in c. A quantitative RealQM treatment of that continuum, from electron screening through to full muon catalysis, is the natural next step.

7 Conclusion

Muon-catalyzed fusion is the deuteron molecular ion with smaller glue. In the RealQM/RealNucleus framework a molecular ion is two nuclei bound by a shared negative charge whose *scale* c sets the bond size, so replacing the electron by the muon — a negative charge some 200$\times$ smaller — contracts the same Coulomb molecule from $\sim$1 Å to $\sim$512 fm — close enough that the residual barrier tunnels and the deuterons fuse. The catalytic cycle and its ceilings (muon lifetime, α-sticking) follow without embellishment, and keep it below break-even. What the picture offers is unity: the same size $\propto c$ law that separates atom from nucleus separates the inert electron molecule from the fusing muon molecule, and puts electron screening and muon catalysis on one continuous scale. The operative variable is size throughout; that one conventionally writes $c = \hbar^2/2m$ only labels the scale by a mass, which belongs, with gravitation, elsewhere.

Code and simulations

The deuteron molecular-ion curve, the $1/m$ rescaling to the muon case, and the underlying $\mathrm{d} + \mathrm{d} \to {}^4\mathrm{He}$ fusion are computed with an open-source WebGPU RealQM solver, runnable from the interactive gallery [10] at `https://claes542.github.io/RealMolecule/gallery.html`; see the deuteron (`nucleus_2h.html`), the $\mathrm{D}+\mathrm{D}$ fusion sweep (`nucleus_DD_fusion_Rsweep.html`), the muon-catalyzed fusion cycle (`muon_catalyzed_fusion.html`), and the electron-versus-muon glue comparison (`muon_vs_deuteron.html`).

Acknowledgements

The physical argument and model are the author's; the numerical implementation was developed and run with AI assistance (Claude, Anthropic).

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