Magnetism in RealQM: Currents, the Nuclear Electron, and the Spin Residue

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Abstract

RealQM reformulates quantum mechanics as multiphase continuum mechanics in real three-dimensional space: electrons are real-valued charge densities on non-overlapping domains with free boundaries, ground states minimising the Coulomb energy. In that form the theory is deliberately real, and produces no electric current — a real wavefunction has $\mathbf{J} = \frac{q}{m}\,\mathrm{Im}(\psi^*\nabla\psi) \equiv 0$ — so magnetism is absent by construction. We develop magnetism as a minimal, current-bearing extension and follow it, honestly, to where it stops. Four results and one boundary. *(1)* A complex, unitary, time-dependent RealQM on the same domains carries a well-defined charge current; the Bernoulli free boundary automatically conserves charge per domain, so nothing new must be assumed, and the Madelung hydrodynamic reading becomes a physical flow in real 3D space for any number of electrons. *(2)* A circulating charge density carries a magnetic moment $\mu_z = -m\mu_B$, quantised by the winding number (gyromagnetic $g = 1$), and reacts correctly to a field: a numerical solver conserves norm and reproduces the Zeeman splitting $2\mu_B B$ to four digits. *(3)* The electron confined in a nucleus is computed as a *flat, real* density, hence currentless, hence carries *no* moment — so nuclear moments come out at nuclear-magneton rather than Bohr-magneton scale, dissolving the classical magnetic-moment objection to nuclear electrons; the same flatness that makes the electron's mass irrelevant makes its moment vanish. *(4)* A single-electron spinor structure delivers $g = 2$: from $(\boldsymbol{\sigma} \cdot \mathbf{D})^2 = \mathbf{D}^2 - q\,\boldsymbol{\sigma} \cdot \mathbf{B}$ (verified numerically) the $g = 2$ term *emerges*, and an $L = 0$ electron splits into two levels with no middle — Stern–Gerlach — non-relativistically. *The boundary:* the many-electron extension fails. A closed shell comes out paramagnetic, not diamagnetic; forcing the spins antiparallel geometrically costs energy (a spinor domain wall), so geometry favours parallel. Closed-shell diamagnetism, and ferromagnetism, require an exchange term that geometric non-overlap does not supply. The precise statement earned is that RealQM's non-overlap reproduces the *spatial* consequences of antisymmetry but not its *spin* consequences: single-particle magnetism is delivered; collective magnetism is not.

Keywords: RealQM; magnetism; charge current; magnetic moment; nuclear electron; $g = 2$; Stern–Gerlach; exchange; closed shell; spinor.

1 Introduction

RealQM [1] represents the N electrons of an atom or molecule by real-valued one-electron functions $\psi_i : \mathbb{R}^3 \to \mathbb{R}$ on non-overlapping domains Ω_i with free (Bernoulli) boundaries, the ground state minimising the Coulomb energy

$$E = \sum_i \tfrac{1}{2}\int_{\Omega_i} |\nabla\psi_i|^2 + \tfrac{1}{2}\iint \frac{\rho(\mathbf{x})\rho(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}\, d\mathbf{x}\, d\mathbf{y}, \qquad \rho = \sum_i q_i\psi_i^2, \tag{1}$$

found by imaginary-time (parabolic) relaxation. This form is validated across chemistry, but it is deliberately real, and a real wavefunction carries no current: with ψ_i real, $\mathrm{Im}(\psi_i^*\nabla\psi_i) = 0$. Magnetism — radiation, moments, susceptibility — is charge in motion, and is therefore absent from the base theory by construction.

This paper builds magnetism as a minimal extension and reports, without embellishment, both what it delivers and where it stops. Sections 2–5 give four results: the current-bearing formulation, orbital magnetism and its field response, the vanishing moment of the nuclear electron, and the single-electron $g = 2$. Section 6 gives the boundary: the many-electron (closed-shell) extension fails, and we state precisely why. Throughout we set $\hbar = 1$ in formulae; masses m_i and charges $q_i = \pm 1$ are kept explicit. Numerical claims are from small, self-contained solvers documented with the source.

2 A current-bearing RealQM

Let each electron function now be complex and time-dependent, $\psi_i : \Omega_i \times \mathbb{R} \to \mathbb{C}$, with charge density and current

$$\rho_i = q_i|\psi_i|^2, \qquad \mathbf{J}_i = \frac{q_i}{m_i}\,\mathrm{Im}(\psi_i^*\nabla\psi_i) = \rho_i\mathbf{v}_i, \quad \mathbf{v}_i = \frac{\nabla S_i}{m_i}, \tag{2}$$

the last form (writing $\psi_i = R_i e^{iS_i}$) being the Madelung reading: current is charge density times a velocity set by the phase gradient. The evolution is unitary on each domain,

$$i\,\partial_t\psi_i = \mathcal{H}_i\psi_i, \qquad \mathcal{H}_i = -\frac{1}{2m_i}\nabla^2 + V_i, \tag{3}$$

with V_i the same real (kernel plus inter-domain Hartree) potential as in the ground-state problem, and the free boundary carrying the *same* Bernoulli condition $\partial_n \psi_i = 0$ on Γ_i.

The free boundary conserves charge. From (3) with real V_i follows continuity, $\partial_t \rho_i + \nabla \mathbf{J}_i = 0$. The rate of change of charge on Ω_i is the boundary flux $-\oint_{\Gamma_i} \mathbf{J}_i \cdot \mathbf{n}$, and the normal current is $\mathbf{J}_i \cdot \mathbf{n} = (q_i/m_i)\,\mathrm{Im}(\psi_i^* \partial_n \psi_i)$, which the Bernoulli condition $\partial_n \psi_i = 0$ sets to zero. Hence $\dot{Q}_i = 0$: *each electron keeps its unit charge under the unitary evolution*, because the boundary that defines the RealQM domains is also a no-leak condition. The static free boundary is exactly what the dynamics needs.

Madelung, physical in 3D. Equations (2) are the standard quantum probability current and the Madelung (1927) hydrodynamic reading. In standard quantum mechanics this reading is physical only for one particle; for N electrons the fluid lives on $3N$-dimensional configuration space. Because RealQM keeps every ψ_i in real $\mathbb{R}^3$, each $(\rho_i, \mathbf{v}_i, \mathbf{J}_i)$ is a genuine three-dimensional density, velocity and current — one physical charge fluid per electron. RealQM thus gives the Madelung flow physical existence in real space for any N (with the caveat that this is RealQM's own fluid, non-overlap standing in for antisymmetry, not a projection of the standard configuration-space object).

Implementation. The dynamics is implemented in Cartesian form $\psi_i = u_i + i v_i$, evolved by the coupled leapfrog $\partial_t u_i = \mathcal{H}_i v_i$, $\partial_t v_i = -\mathcal{H}_i u_i$ (Visscher staggered, norm-preserving to $O(\Delta t^2)$). The polar form $R_i e^{iS_i}$ is avoided: it is singular at nodes (S_i undefined, $\mathbf{v}_i$ divergent where $R_i \to 0$) — exactly the axis of a winding state — whereas (u_i, v_i) is smooth through nodes and linear.

3 Orbital magnetism and its response to a field

A stationary state with a spatial phase winding, $\psi = R(\mathbf{x}) e^{im\phi}$, carries an azimuthal current and a magnetic moment

$$\boldsymbol{\mu} = \tfrac{1}{2} \int \mathbf{r} \times \mathbf{J}\, d^3x, \qquad \mu_z = -m\, \mu_B, \tag{4}$$

quantised in the winding number and *independent of the density's shape*: the moment-arm ($\propto s$) and the winding velocity ($\propto 1/s$) cancel, which is the gyromagnetic relation $\boldsymbol{\mu} = (q/2m)\mathbf{L}$ with $L_z = m$ fixed topologically by single-valuedness of $e^{im\phi}$. This has gyromagnetic factor $g = 1$.

Numerical validation. A Cartesian (u, v) leapfrog for one electron in a 2D harmonic well, initialised in the $m = 1$ winding state $\psi = (x + iy)e^{-r^2/2}$, conserves norm to 1.000000 and holds $\mu_z/\mu_B = -1.00$ (grid value -0.997) stable over 4000 steps. The circulating density is genuine.

Reaction to a magnetic field. Adding the field by minimal coupling $\mathbf{p} \to \mathbf{p} - q\mathbf{A}$ (uniform B along z, $\mathbf{A} = \frac{B}{2}(-y, x)$) and evaluating the energy of the $m = \pm 1$ states gives the Zeeman response (Table 1): the circulating states shift linearly, splitting $E(+1) - E(-1) = 2\mu_B B$ to four digits, while the non-circulating $m = 0$ state barely moves (weak diamagnetism only). So a RealQM circulating charge density reacts to a field exactly as a moment $\pm\mu_B$ should — ordinary orbital magnetism, from a charge density in real space, no spin invoked.

B	$E(m{=}0)$	$E(+1) - E(-1)$	$2\mu_B B$
0.05	0.999	+0.0499	0.0500
0.10	1.000	+0.0999	0.1000
0.20	1.004	+0.1998	0.2000

Table 1: Zeeman response of the circulating density (minimal coupling). The $m = \pm 1$ splitting matches $2\mu_B B$; the $m = 0$ (non-circulating) state is inert to linear order.

A real route: magnetism needs a velocity, not the complex plane. It is worth separating what in the above is genuinely "complex" from what is mere bookkeeping, because the distinction pins down exactly how much of the quantum enters. From the density *alone* — a single real field $\psi = R$ — no magnetism can come, since $\mathbf{J} = q\,\mathrm{Im}(\psi^*\nabla\psi) \equiv 0$: a lone amplitude carries no motion, and a magnetic moment is charge in motion. But complex numbers are not the only way to carry a second field. Continuum mechanics never uses them, yet always carries *two* real fields, a density ρ and a velocity $\mathbf{v}$; carry the same pair here and the whole of orbital magnetism is real. The current is $\mathbf{J} = \rho\,\mathbf{v}$, a real circulating flow ($\nabla\cdot\mathbf{J} = 0$ in a stationary state); the flow costs a real kinetic energy

$$T_{\text{flow}} = \int \tfrac{1}{2}\,\rho\,|\mathbf{v}|^2 = \int \frac{|\mathbf{J}|^2}{2\rho}\,d^3x, \tag{5}$$

it carries the real angular momentum $L_z = \int \rho\,(\mathbf{r}\times\mathbf{v})_z\,d^3x$ and the ordinary magnetostatic moment $\boldsymbol{\mu} = \frac{1}{2}\int \mathbf{r}\times\mathbf{J}\,d^3x$ of (4), and the magnetic state is obtained by minimising the static RealQM energy *augmented by* (5) at fixed L_z (or, in a field, minimising $E - \boldsymbol{\mu}\cdot\mathbf{B}$). No i enters any of this: it is a real charge vortex, precisely the $(\rho, \mathbf{v})$ of continuum mechanics an ångström across. The complex $\psi = \sqrt{\rho}\, e^{iS}$ used in (2) and in the leapfrog above is simply these two real fields repackaged into one symbol, with $\mathbf{v} = \nabla S/m$; the phase *is* the velocity potential — which is why the Cartesian (u, v) integrator is the honest one and the polar form only a reading.

The one thing the real $(\rho, \mathbf{v})$ theory cannot supply on its own is the *integer*. Single-valuedness of e^{iS} — the phase is an *angle*, defined mod 2π — forces the quantised circulation $\oint \mathbf{v} \cdot d\boldsymbol{\ell} = 2\pi m/m_e$, $m \in \mathbb{Z}$, and with it $L_z = m$ and the $\mu_z = -m\mu_B$ of (4); a purely real formulation would have to impose this as a separate quantisation condition (the Onsager–Feynman vortex quantisation of real superfluids). What is unavoidable there is the *circle* — the periodicity of an angle — not the complex plane as such. Magnetism's dynamics, energy and moment are real continuum mechanics; only the quantum integer m needs the periodic phase.

4 The nuclear electron carries no moment

The classical objection that retired the proton–electron nucleus in 1932 was magnetic: an electron confined inside a nucleus should carry a moment of order $\mu_B = 1836\, \mu_N$, whereas nuclear moments (the neutron's $-1.913\, \mu_N$ included) are of order μ_N — a thousandfold too small. In a charge-density theory the moment is the current's (4), and a real density has $\mathbf{J} = 0$. RealNucleus computes the deuteron's confined electron as a *flat, real, node-less* density [2]; it is the $m = 0$, currentless case, and carries *no* magnetic moment. Nuclear moments are then set by the protons and come out at μ_N scale. The thousandfold overshoot does not occur, because in a charge-density theory there is no intrinsic moment to carry — only the current, and the flat electron's current is zero.

Energetics, and one flatness for two objections. A winding at confinement scale σ costs $\Delta T = \frac{\hbar^2}{2m_e} m^2 \langle 1/s^2 \rangle \approx \mathrm{Ry}\, (a_0/\sigma)^2$: about one Rydberg (27 eV) for a free electron ($\sigma \sim a_0$) but $\sim$GeV for a femtometre-caged one — thousands of times the deuteron binding, so the caged electron cannot afford a winding and settles at $m = 0$. A radial relaxation confirms this self-consistently. And the point is the *same* flatness that was introduced to make the electron's *mass* irrelevant to nuclear binding (a flat density has vanishing kinetic gradient): that flatness, being real and currentless, makes the *magnetic moment* vanish too. Two of the classical objections to nuclear electrons — confinement energy and magnetic moment — are answered by one property of the confined-electron solution.

5 Single-electron spin: $g = 2$ from a spinor structure

Orbital circulation gives $g = 1$; the electron's spin moment has $g = 2$ (a spin of $\hbar/2$ yields a full μ_B), and the two-valued Stern–Gerlach response is not orbital at all. This is not a demand for relativity. By Lévy-Leblond [4], $g = 2$ follows from a first-order, *non-relativistic* spinor equation. Equipping the domains with a two-component spinor and the linearised structure $(\boldsymbol{\sigma} \cdot \mathbf{D})$, $\mathbf{D} = \mathbf{p} - q\mathbf{A}$, the operator identity

$$(\boldsymbol{\sigma} \cdot \mathbf{D})^2 = \mathbf{D}^2 \nvdash - q\,\boldsymbol{\sigma} \cdot \mathbf{B} \tag{6}$$

holds (verified numerically to grid accuracy, residual $\sim 10^{-3}$), so the $g = 2$ Zeeman term $-q\,\boldsymbol{\sigma} \cdot \mathbf{B}$ *emerges* from squaring $\boldsymbol{\sigma} \cdot \mathbf{p}$ — it is not inserted. Consequently an $L = 0$ (orbitally inert) electron — the state a scalar density leaves undeflected — splits under the spinor into *two* levels $\pm\mu_B B$, splitting $2\mu_B B$, $g_s = 2$, with *no middle*, against the three-level ($m = -1, 0, +1$) orbital pattern. This is Stern–Gerlach, delivered non-relativistically: spin reaches $\pm\mu_B$ at $\pm\frac{1}{2}$ unit of angular momentum ($g = 2$, no middle) where orbital needs ± 1 unit ($g = 1$, with middle). The "2" is the SU(2) double cover of the rotation group — geometric, not relativistic.

6 The boundary: collective magnetism is not supplied

Here the extension stops, and it is important to say so plainly. "Spin" bundles two logically distinct things: *statistics-spin* (the fermionic exclusion — two electrons per orbital, the periodic table), which RealQM realises geometrically by non-overlap with no spin variable; and *magnetic-spin* (the two-valued moment of §5). The single-electron magnetic-spin is delivered. The many-electron case is not.

The closed-shell test fails. A filled shell must be diamagnetic — two electrons pairing to zero net moment. We tested the spin sector of the naive extension: two electrons, each with an independent spinor, in a field. The ground state is the aligned pair with net moment $2\mu_B$ —

paramagnetic, not diamagnetic. Nothing in geometric non-overlap forces the two spins antiparallel.

Geometry does not rescue it. One might hope continuity of the spinor field across the shared domain boundary would force the spins antiparallel. It does the opposite. Forcing the joined field antiparallel means a spinor *domain wall*, whose kinetic cost is $T_{\rm spin} = \frac{1}{8}\int(\theta')^2 \approx \pi^2/24w > 0$, while the parallel configuration costs nothing: continuity *favours parallel* (paramagnetic). And in the actual RealQM structure — separate functions on non-overlapping domains — the spinors are simply uncoupled and degenerate, aligning with any field. Either way geometry supplies no antiparallel pairing. Obtaining diamagnetism requires an antiparallel-pairing (exchange) term $J\,\mathbf{S}_1\cdot\mathbf{S}_2$, which is a spin-statistics ingredient non-overlap does not carry. The same missing exchange is what drives *collective* magnetism generally: ferromagnetism is the spontaneous alignment of atomic moments by exchange, and it too is absent.

The precise statement. Geometric non-overlap reproduces the *spatial* consequences of antisymmetry (exclusion, shells, the whole of chemistry) but not its *spin* consequences (singlet pairing, exchange splitting, closed-shell diamagnetism, ferromagnetism). Single-particle magnetism is delivered; collective, exchange-driven magnetism is a genuine limitation, not a gap awaiting a boundary condition.

7 Interpretive frame: magnetism without relativity

The results sit naturally in a reading of RealQM in which an electron is not a standalone object but a *domain of a configuration*, its size fixed by Coulomb balance within the configuration rather than by any intrinsic scale [3]. On that reading the single-atom magnetic moment is a configuration property — μ_B when the electron is loosely confined (a winding it can afford), none when tightly caged — and no rest-energy or Compton scale is invoked anywhere; even $g = 2$ is the geometric SU(2) factor, not a relativistic one. This frame is not needed for the computational results of §§2–6, which stand on their own; it is offered as the picture that makes them coherent, and as the reason the one true residue — collective magnetism — is a matter of missing *exchange*, not of missing relativity.

8 Conclusion

Magnetism enters RealQM as a minimal current-bearing extension: complex, unitary dynamics on the same domains, with the free boundary conserving charge and the Madelung flow made physical in real 3D. From it, orbital magnetism is delivered and reacts correctly to a field; the flat confined electron carries no moment, dissolving the magnetic-moment objection to nuclear electrons by the same flatness that removes its mass; and a single-electron spinor structure yields $g = 2$ and Stern–Gerlach non-relativistically. The extension then reaches a firm boundary: closed-shell diamagnetism and ferromagnetism require an exchange term that geometric non-overlap does not supply, because non-overlap carries the spatial but not the spin consequences of antisymmetry. What RealQM delivers is single-particle magnetism, honestly and without relativity; what it does not deliver is the collective, exchange-driven magnetism that binds many moments together — and locating that boundary precisely is itself a result.

The oscillating-current partner of this circulating magnetism — radiation and atomic spectra — is treated in the companion [5], on the shared complex time-dependent foundation.

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