Complex Time-Dependent RealQM: Currents, Radiation, and Magnetism from the Real-Space Continuum

Claes Johnson[1]

[1]KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden ,
claesjohnson@gmail.com

July 25, 2026

Text and argument by the author; numerical computations with AI assistance (Claude, Anthropic).

Abstract

RealQM in its established form is a *real-valued*, *parabolic* theory: electrons are real-valued charge densities $\psi_i : \mathbb{R}^3 \to \mathbb{R}$ on non-overlapping domains, and ground states are found by imaginary-time (gradient-flow) relaxation of the Coulomb energy. This form is validated across chemistry, but it produces no electric *current* — a real wavefunction has $\mathbf{J} = \frac{\hbar}{m}\mathrm{Im}(\psi^*\nabla\psi) \equiv 0$ — and therefore no radiation currents, no magnetic moments, and no spin observables; the base theory itself flags these as requiring a separate extension. Here we formulate that extension: a *complex*, *unitary*, time-dependent RealQM, $i\hbar\,\partial_t\psi_i = \mathcal{H}_i\psi_i$ on each domain, carrying a well-defined charge current, alongside the existing parabolic relaxation. We show that the free-boundary (Bernoulli / zero-flux) condition that already defines the domains *automatically* makes the unitary evolution charge-conserving on each domain, so the extension respects the multiphase structure with no new assumption. Two kinds of motion then generate the electromagnetic observables the real theory cannot: *oscillation* (a superposition of states, a time-varying density) sources dipole radiation, and *circulation* (a spatial phase winding, a standing current) gives a magnetic moment $\boldsymbol{\mu} = \frac{1}{2}\int \mathbf{r} \times \mathbf{J}$. Ground states are real, hence currentless, hence carry no moment — which is why RealQM's flat confined electron has none — while excited or winding states carry both. We are explicit that this is a proposed formulation, consistent with but extending the validated real theory: the ground-state energetics are unchanged; the current-bearing dynamics, and its use for the free electron's own moment and for spin, are new and open.

Keywords: RealQM; time-dependent; complex wavefunction; charge current; free boundary; radiation; magnetic moment; spin; parabolic vs unitary.

1 Why an extension is needed

RealQM [1] represents the N electrons of an atom or molecule by real-valued one-electron wave functions $\psi_i : \mathbb{R}^3 \to \mathbb{R}$ supported on non-overlapping domains Ω_i with free (Bernoulli) boundaries, and finds the ground state by minimising the Coulomb energy

$$E \;=\; \sum_i \tfrac{1}{2}\int_{\Omega_i} |\nabla\psi_i|^2 \;+\; \tfrac{1}{2}\iint \frac{\rho(\mathbf{x})\rho(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}\,d\mathbf{x}\,d\mathbf{y}, \qquad \rho = \sum_i q_i\,\psi_i^2, \tag{1}$$

via imaginary-time relaxation $\dot{\psi}_i + \mathcal{H}_i\psi_i = 0$, a structurally *parabolic* (dissipative) flow toward the lowest state. This is the validated core, and it is deliberately real.

A real wavefunction, however, carries *no current*: with ψ_i real, $\mathrm{Im}(\psi_i^*\nabla\psi_i) = 0$. So the base theory cannot express any observable that is built from charge *in motion* — dipole radiation, magnetic moments, magnetic susceptibility, NMR shifts, hyperfine structure. RealQM says as much of itself, flagging these as outside its current form and requiring “a separate extension.” This note writes that extension down.

The guiding principle is minimal: keep the domains, the Coulomb energy (1), and the free-boundary condition exactly as they are, and add only what is needed for motion — a *complex* phase and a *unitary* time law that carries it.

2 The complex time-dependent formulation

In the formulation below we set $\hbar = 1$; the masses m_i and charges $q_i = \pm 1$ are kept explicit, since different domains may carry different mass (electrons, protons, or the equal-mass device of the nuclear regime), and the current depends on that mass.

Let each electron wave function now be complex and time-dependent, $\psi_i : \Omega_i \times \mathbb{R} \to \mathbb{C}$, with charge density and charge current

$$\rho_i = q_i\,|\psi_i|^2, \qquad \mathbf{J}_i = \frac{q_i}{m_i}\,\mathrm{Im}(\psi_i^*\nabla\psi_i)\,, \qquad \rho = \sum_i \rho_i, \quad \mathbf{J} = \sum_i \mathbf{J}_i. \tag{2}$$

The current (2) has a transparent hydrodynamic (Madelung) reading. Writing $\psi_i = R_i\,e^{iS_i}$ with real amplitude R_i and real phase S_i, one has $\psi_i^*\nabla\psi_i = R_i\nabla R_i + i\,R_i^2\,\nabla S_i$, so $\mathrm{Im}(\psi_i^*\nabla\psi_i) = |\psi_i|^2\,\nabla S_i$ and

$$\mathbf{J}_i \;=\; \rho_i\,\mathbf{v}_i, \qquad \mathbf{v}_i = \frac{\nabla S_i}{m_i}. \tag{3}$$

The current is charge density times a *velocity field* $\mathbf{v}_i$ set by the phase gradient ($\mathbf{p}_i = \nabla S_i$ is the local momentum, $\mathbf{v}_i = \mathbf{p}_i/m_i$). Two facts follow at once: a *real* density has constant phase, $\nabla S_i = 0$, hence $\mathbf{v}_i = 0$ and $\mathbf{J}_i = 0$ — no motion, no current; and a phase that *winds* in space, $S_i = m\phi$, gives an azimuthal velocity $|\mathbf{v}_i| = m/(m_i s)$ and a standing circulation. The mass enters only through $\mathbf{v}_i = \nabla S_i/m_i$: it scales the current when there *is* a phase gradient, and is irrelevant when there is none — consistent with the flat electron being both mass-irrelevant and currentless.

Madelung, made physical in three dimensions. Equations (2)–(3) are nothing exotic: they are the standard quantum probability current and the Madelung (1927) hydrodynamic reading of the Schrödinger equation — ρ a density, $\mathbf{v} = \nabla S/m$ a velocity field. But in *standard* quantum mechanics that reading is physical only for a *single* particle; for N electrons the wave function lives on $3N$-dimensional configuration space, and the Madelung fluid lives there too, with no existence in physical three-dimensional space. RealQM removes exactly this obstacle: because its N electrons are one-electron densities ψ_i on domains in real $\mathbb{R}^3$, each $(\rho_i, \mathbf{v}_i, \mathbf{J}_i)$ is a genuine three-dimensional density, velocity, and current — one physical charge fluid per electron, on its own territory. **RealQM thus gives the Madelung hydrodynamic picture physical existence as a real-space flow, for any number of electrons, which standard quantum mechanics**

confines to configuration space. The sense of this must be stated carefully: RealQM is not standard N-body QM (non-overlap replaces antisymmetry), so this is RealQM's *own* Madelung fluid within its own real-space formulation, not a projection of the standard configuration-space fluid down into 3D. Within that formulation, however, the flow is as physical as the charge density RealQM already takes ψ_i^2 to be.

The evolution is *unitary* on each domain,

$$i\,\partial_t\psi_i \;=\; \mathcal{H}_i\,\psi_i, \qquad \mathcal{H}_i = -\frac{1}{2m_i}\nabla^2 + V_i, \tag{4}$$

with V_i the same real (kernel plus inter-domain Hartree) potential as in the ground-state problem, $V_i(\mathbf{x}) = W(\mathbf{x}) + \sum_{j\neq i}\int q_j|\psi_j|^2/|\mathbf{x}-\mathbf{y}|\,d\mathbf{y}$. The free boundary carries the *same* Bernoulli condition as before,

$$\frac{\partial\psi_i}{\partial n} = 0 \quad \text{on } \Gamma_i = \partial\Omega_i. \tag{5}$$

The free boundary conserves charge automatically. Equations (4)–(5) have a property that makes the extension consistent with the multiphase structure without any new assumption. From (4) with real V_i follows the continuity equation

$$\partial_t\rho_i + \nabla\cdot\mathbf{J}_i = 0 \quad \text{in } \Omega_i. \tag{6}$$

Integrating over Ω_i and using the divergence theorem, the rate of change of the charge on domain i is the flux through its boundary, $\dot{Q}_i = -\oint_{\Gamma_i}\mathbf{J}_i\cdot\mathbf{n}\,dS$. But the normal current is $\mathbf{J}_i\cdot\mathbf{n} = (q_i/m_i)\,\mathrm{Im}(\psi_i^*\,\partial_n\psi_i)$, and the Bernoulli condition (5) sets $\partial_n\psi_i = 0$, so

$$\mathbf{J}_i\cdot\mathbf{n} = 0 \text{ on } \Gamma_i \qquad \Longrightarrow \qquad \dot{Q}_i = 0. \tag{7}$$

Each electron keeps its unit charge under the unitary evolution, because the very boundary condition that defines the RealQM domains is also a zero-current (no-leak) condition. The free boundary, introduced for the statics, is exactly what the complex dynamics needs to remain a clean N-electron partition.

Are the domains held fixed? In the minimal formulation adopted here, yes: the domains Ω_i are kept at their ground-state shape and each ψ_i evolves *within* its fixed domain under (4), subject to the zero-flux boundary condition (5)–(7) that conserves its charge. This is exact for small-amplitude motion, and — importantly for the applications here — it is well-justified for a *winding* state, because a winding $\psi_i = R_i\,e^{im\phi}$ changes only the *phase* of the density, not its magnitude: $|\psi_i|^2 = R_i^2$ is essentially the ground-state density (with, at most, a node forced onto the winding axis), so the free boundary, which is fixed by $|\psi_i|^2$, barely moves. The magnetic moment is exactly such a winding state, so holding the domains fixed is not a crude approximation but a controlled one. The general, large-amplitude case — in which the free boundary Γ_i co-evolves with the ψ_i, the domains genuinely moving — is the harder problem, and its well-posedness is listed as open (§5). What must not be lost is that in RealQM the partition is normally a *primary* variational degree of freedom; fixing it is a deliberate small-amplitude simplification, exact at rest and controlled for windings, not a claim that the domains are permanently rigid.

Relation to the parabolic form. The two dynamics are the same operator under $\tau = it$: the parabolic relaxation $\dot{\psi}_i = -\mathcal{H}_i\psi_i$ drives ψ_i to the real ground state (imaginary time), while the unitary law (4) carries the complex phase (real time). *Relaxation finds the state; unitary evolution carries the current.* A system prepared in its ground state and left alone has real ψ_i, hence $\mathbf{J} = 0$: no radiation, no moment, at rest — as it should be.

3 The electromagnetic observables the real theory could not give

With a current in hand, the observables follow by the classical (Schrödinger–Maxwell) reading: the charge density and current $\rho, \mathbf{J}$ source the electromagnetic field, and every electromagnetic observable is a functional of them. Two distinct motions matter.

Oscillation $\Rightarrow$ radiation. Radiation, in RealQM, is not a "superposition of stationary states" — that phrase is configuration-space language for *setting up* a time-dependence. Physically it is the *charge density itself oscillating in real space*: a charge cloud that sloshes back and forth is an antenna, and it radiates. A non-stationary configuration has a time-varying density $\rho(\mathbf{x}, t)$ and an oscillating dipole $\mathbf{d}(t) = \int \mathbf{x}\,\rho\,d\mathbf{x}$, which radiates at its oscillation frequency ω by the Larmor formula, exactly as a classical moving charge does — with the current (2) supplying the physical charge flow behind the motion. (One way to *arrange* such an oscillation is to start the state between two stationary ones, $\psi_i = a\phi_1 e^{-iE_1 t} + b\phi_2 e^{-iE_2 t}$, giving $\omega = E_2 - E_1$; but the physics is the charge moving in space, not the abstract superposition.) The atomic-spectrum consequences — the levels, the selection rules read as geometry, and helium's lines — are developed in the companion [3].

Circulation $\Rightarrow$ magnetic moment. A stationary state with a *spatial phase winding*, $\psi_i = R(\mathbf{x})\,e^{im\phi}$ about an axis, carries a standing azimuthal current and a magnetic moment

$$\boldsymbol{\mu} \;=\; \tfrac{1}{2}\int \mathbf{r}\times\mathbf{J}\,d^3x, \qquad \mu_z = -\,m\,\mu_B \quad \text{(for a normalised electron density)}, \tag{8}$$

the moment being quantised in the winding number m and independent of the density's shape. A real ($m = 0$) density gives $\mu = 0$. This is the magnetostatic partner of the oscillation: charge going in circles rather than back and forth.

Why the confined electron has no moment. The two facts combine into the result of the companion note [2]: RealQM computes the deuteron's caged electron as a *flat, real* density — the $m = 0$, currentless case — so it carries no magnetic moment, and nuclear moments come out at nuclear-magneton scale. That result lives already in the *real* theory (ψ real $\Rightarrow$ $\mathbf{J} = 0$); the present extension is what would be needed for the *nonzero* moments (the free electron's own μ_B, the neutron's residual), which require a genuine current.

4 Spin as a further structure

Spin is not a primary variable of RealQM; Pauli exclusion is realised geometrically by non-overlap. The natural place for spin in the complex extension is an *internal circulation*: the free electron's μ_B cannot be an ordinary orbital winding (the free/atomic ground state is real, $m = 0$, with zero orbital moment), so if it is a current at all it is a Compton-scale internal circulation — the Zitterbewegung structure — carrying angular momentum $\frac{1}{2}$ and moment $\sim \mu_B$. Equivalently, one may add the $\pm\frac{1}{2}$ domain label the base theory suggests, with its flux balance entering the magnetic field. Either route is a genuine addition beyond (4); we flag it, and the anomalous $g/2 = 1.00116\ldots$, as open (§5) rather than claim them here.

5 What is proposed, what is validated, what is open

Validated (unchanged): the real, parabolic ground-state RealQM and all its chemistry — energies, geometries, reactions, condensed phases — are untouched. The extension reduces to it exactly for a system at rest (real ψ, $\mathbf{J} = 0$).

Proposed here: the complex, unitary, time-dependent law (4) on the RealQM domains, its current (2), and the observation (7) that the Bernoulli free boundary makes it charge-conserving per domain. This gives radiation and (orbital) magnetic moments a first-principles home in the real-space continuum.

Open:

- **The moving free boundary.** For large-amplitude dynamics the domains Ω_i should co-evolve with the ψ_i; the well-posedness of the coupled complex-field / free-boundary problem is a genuine mathematical question. The fixed-domain version is the clean small-amplitude case.

- **The free electron's μ_B and spin.** These require the Compton-scale internal circulation of §4, not the atomic-scale winding (8); they are not delivered by (4) alone.

- **The anomalous moment.** $g/2 = 1.00116\ldots$ (QED's most precise number) would be the sharp test of any current-based account of the electron moment, and is untouched here.

- **Numerics.** Implement (4) in *Cartesian* form, $\psi_i = u_i + i\, v_i$ with two real fields, evolved by the coupled leapfrog $\partial_t u_i = \mathcal{H}_i v_i$, $\partial_t v_i = -\mathcal{H}_i u_i$ (staggered by $\frac{1}{2}$ step, norm-preserving to $O(\Delta t^2)$). The *polar* form $\psi_i = R_i e^{iS_i}$ must be avoided in computation: it is singular at nodes (S_i undefined and $\mathbf{v}_i = \nabla S_i/m_i$ divergent where $R_i \to 0$), which is exactly the axis of a winding state; the Cartesian representation is smooth through nodes and linear. Read the current (2) and the moment (8) off (u_i, v_i) directly. This sits on top of the existing solver and is the vehicle for the free-vs-caged moment test.

6 Conclusion

RealQM was built real and parabolic, and is validated there; but charge in motion — radiation, magnetism, spin — lives only in a complex, unitary, time-dependent form, which the base theory itself marks as missing. The extension is minimal: the same domains, the same Coulomb energy, the same Bernoulli boundary, now with a complex phase evolved by $i\,\dot{\psi}_i = \mathcal{H}_i\psi_i$. The free boundary turns out to be exactly the zero-current condition that keeps the evolution a clean N-electron partition, so nothing new must be assumed. Oscillation then gives radiation and circulation gives a magnetic moment, while the real ground state — currentless — gives neither, which is why the flat confined electron carries no moment. What remains open, honestly, is the free electron's own μ_B at the Compton scale, the anomalous g-factor, the moving-boundary dynamics, and the numerics — but the formulation on which all of these can now be posed is in place.

References

[1] C. Johnson, *Real Quantum Mechanics as Multiphase 3D Continuum Mechanics*, manuscript, 2026; `https://claes542.github.io/RealMolecule/gallery.html`.

[2] C. Johnson, *Magnetism as Circulating Charge: Why the Nuclear Electron Carries No Magnetic Moment*, 2026 (companion note).

[3] C. Johnson, *RealQM Atomic Spectra: Radiation as Real Charge Oscillation*, manuscript, 2026 (companion).