Deterministic RealQM Beta Decay: A Phase-Coincidence Half-Life

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Abstract

Radioactive decay poses two separate questions: *when* a given nucleus decays, and *what carries* the released energy and momentum. Standard quantum mechanics answers the first with irreducible randomness — the decay time is uncaused, only its probability is defined — and the second, for beta decay, with the neutrino. This paper addresses the first question in time-dependent RealQM, where the "neutron" is a proton–electron pair bound by Coulomb alone. Each charge domain carries a phase $\theta_k(t) = -E_k t/\hbar$ fixed by its computed energy, and the loosely bound electron is released when the inter-domain phase-beats fall jointly in an escape window. On the reading of de Broglie and Bohm the decay is then *deterministic*, the randomness epistemic (ignorance of the phases), and the exponential, memoryless law emerges as the ensemble statistics of a coincidence rather than as a signature of fundamental indeterminism. We are equally explicit that the specific coexisting-domain mechanism, traced through the full time-dependent RealQM, does *not* arise: the escaping domain couples to its neighbours only through their phase-invariant densities across zero-flux boundaries, so the phases never reach it and the decay is plain Gamow. Gating would require an added phase-permeable (Josephson) interface, which we do not adopt; the determinism is de Broglie–Bohm, the mechanism is not RealQM's. We are explicit about scope. The energy budget is a distinct question: RealQM conserves energy by construction, and we examine whether the continuous electron spectrum and the recoil momentum can be balanced without a neutrino by the residual charge and the radiated field. A quantitative estimate shows they cannot — the radiated field carries only a percent-level ($\sim \alpha$) share, far short of the $\sim 60\%$ of the released energy the antineutrino carries — so the energy–momentum balance still requires the neutrino. The contribution is therefore a deterministic account of beta-decay *timing*, not an elimination of the neutrino.

1 Two questions, kept apart

Free-neutron beta decay, $n \to p + e^- + \bar{\nu}$, is usually discussed as a single phenomenon, but it puts two logically independent questions to a theory.

When does a given neutron decay? Empirically the answer is a definite half-life (mean life ≈ 880 s) with an exponential, memoryless survival law. Standard quantum mechanics holds the individual decay time to be *irreducibly random* — uncaused, with only a rate λ (from Fermi's Golden Rule) defined; two identical neutrons decay at different times for no reason, and the exponential law is read as the fingerprint of that fundamental indeterminism.

What carries the released energy $Q \approx 0.782$ MeV and the momentum? Here the electron's energy spectrum is *continuous* up to an endpoint at Q, a two-body decay would make it monoenergetic, and the resolution is the neutrino, carrying the balance of energy and momentum.

These are separate axes — *timing* versus *carriers* — and a theory may speak to one without settling the other. This paper is about the first. We work in time-dependent RealQM, where there is no elementary neutron and no weak force: the model "neutron" is a single proton–electron pair bound by the same Coulomb energy as every atom and nucleus, found *unbound*, so its separation into a free proton and electron is already a Coulomb fact [5]. What the static picture leaves open is the *dynamics of timing*: how fast, and whether random. That is what the time-dependent form settles — and it settles it on the side of determinism. On the second question, the carriers, we reach a negative and honestly limiting result (§4).

2 Time-dependent RealQM

RealQM represents a system of charges by unit densities ψ_i^2 on non-overlapping domains $\Omega_i \subset \mathbb{R}^3$, with total charge density $\rho = \sum_i q_i \psi_i^2$ and energy

$$E[\psi] = \sum_i \tfrac{1}{2}\int_{\Omega_i} |\nabla\psi_i|^2 + \tfrac{1}{2}\iint \frac{\rho(x)\rho(y)}{|x-y|}\,dx\,dy \;=\; \sum_k E_k, \tag{1}$$

minimised in the ground state over the ψ_i and the free boundaries $\partial\Omega_i$; the last equality decomposes the total into per-domain energies E_k. The time-dependent form promotes the ψ_i to complex fields evolving by the first-order law generated by (1) [7],

$$i\,\partial_t\psi_i = \frac{\delta E}{\delta\psi_i^*} = -\tfrac{1}{2m_i}\nabla^2\psi_i + V_i[\rho]\,\psi_i, \tag{2}$$

with $V_i[\rho]$ the Coulomb potential felt by domain i. A stationary state of energy E_k then carries the pure phase rotation

$$\psi_k(t) = \psi_k\, e^{-iE_k t/\hbar}, \qquad \theta_k(t) = -E_k t/\hbar. \tag{3}$$

Because (2) is generated by the energy functional (1) as Hamiltonian, with no explicit time dependence, the total energy is conserved exactly, $\frac{d}{dt}E[\psi(t)] = 0$; a radial prototype of the caged proton–electron pair, evolved by a staggered leapfrog, confirms this numerically, the total-energy drift scaling linearly with the time step and extrapolating to zero. We use energy conservation below; the timing result, however, does not depend on it.

3 Deterministic decay

In the standard account the moment a given nucleus decays is *irreducibly random*: the barrier fixes a probability per unit time, but the individual decay time is uncaused. Time-dependent RealQM reads it differently. The decaying configuration is a set of charge domains evolving by the full time-dependent equations — N complex domain fields on moving domains $\Omega_i(t)$,

$$i\hbar\,\partial_t\psi_i = \Big[-\tfrac{\hbar^2}{2m_i}\nabla^2 + q_i\,\phi[\rho]\Big]\psi_i, \quad \rho = \sum_j q_j|\psi_j|^2, \quad -\nabla^2\phi = 4\pi\rho, \tag{4}$$

on non-overlapping domains $\Omega_i(t)$ whose free boundaries are fixed by density balance and are free to move, with the total energy $E = \sum_i \frac{\hbar^2}{2m_i} \int_{\Omega_i} |\nabla \psi_i|^2 + \frac{1}{2} \int \rho\phi$ conserved. This evolution is *deterministic*: given the initial charge configuration, the whole history — the loosely bound electron leaking across the barrier, the free boundary tracking density balance, the domain emptying — is fixed. The apparent randomness of radioactivity, identical-looking nuclei decaying at different times, is then *epistemic*: the nuclei differ in the exact initial configuration one neither knows nor controls, and the ensemble statistics follow from that ignorance alone. This is the deterministic reading of de Broglie and Bohm [2, 3], carried here by the real-space charge density rather than by hidden point positions.

What makes the decay *exponential*, with a definite half-life, is the tunnelling, and the density dynamics supply it directly. A metastable domain behind a barrier leaks at a constant rate: integrating (4) in real time (or its single-domain reduction), the trapped charge decays exponentially, the half-life is read straight off the dynamics, and sweeping the barrier reproduces the Geiger–Nuttall law — all without the semiclassical (WKB) factor, which is thereby validated rather than assumed; the narrow, long-lived resonances are reached exactly by the complex-energy (Siegert) width. The half-life is a property of the barrier the extended charge must cross; determinism governs only *when* a given configuration decays, not the rate.

On a phase-coincidence mechanism. One is tempted to seek something sharper — coexisting domains carrying phase clocks $e^{-iE_it/\hbar}$, the escape gated by their coincidence. We record, to be exact, that this does *not* follow from RealQM. In (4) the escaping domain couples to its neighbours only through their *densities* $|\psi_j|^2$, which are phase-invariant, and the free boundaries carry zero flux; so the neighbouring phases never reach it, and the moving free boundary — which does move, tracking density balance — transmits density, not phase. Phase-coincidence gating would require a phase-permeable (Josephson) interface, $\sum_j K_{ij}\psi_j|_{\Gamma_{ij}}$ replacing the zero-flux condition, an addition the variational free boundary does not provide — and one that is in any case unnecessary, the decay and its half-life being already carried by the density dynamics. We do not invoke it.

It is worth being explicit about scope. What the full time-dependent RealQM supplies is a *deterministic, real-space* account of the decay — charge tunnelling through the Coulomb barrier, the whole history fixed by the initial configuration — not a rate law beyond barrier penetration itself. The timing statistics and the half-life are those of Gamow tunnelling, recast as deterministic charge-density dynamics rather than irreducible chance; the gain is ontological (and the refutation of a spurious gating mechanism), not a new number.

4 The energy and momentum budget: the neutrino is not removed

The timing result says nothing about *what carries* the released energy and momentum, and it is essential not to overreach here. One might hope that, energy being conserved by construction (§2), the continuous electron spectrum is simply a phase-dependent *partition* of the conserved E_0 among the electron, the recoiling residual charge, and the radiated field,

$$E_0 = E_e + E_{\text{rec}} + E_{\text{field}}, \tag{5}$$

with no neutrino required. We tested this quantitatively, and it does not hold.

Three facts settle it. First, the shares are lopsided: in free-neutron decay the electron's *mean* kinetic energy is only about 0.30 MeV of the $Q \approx 0.78$ MeV released, the daughter recoil is negligible (below a keV, the proton being heavy), and the remaining ≈ 0.48 MeV — the *majority* of the budget — is what the antineutrino carries. Second, a two-body $e + p$ release cannot supply this: momentum conservation from rest makes the products exactly back-to-back, and because kinetic energy is $p^2/2m$ the heavy proton takes essentially none, so the electron would carry nearly all of Q — a line at the endpoint, not a continuum. A two-dimensional solve of (2) for the electron and proton confirms both the exact back-to-back emission and the conservation of total momentum to machine precision. Third, the radiated field is far too weak to make up the difference: estimating the Larmor dipole radiation from the electron's own current in that solve gives a radiated energy fraction of order α — a percent at the relevant velocities, consistent with the known inner-bremsstrahlung level — whereas the missing share is $\sim 60\%$. The field falls short by roughly two orders of magnitude, and the field momentum, bounded by E_{field}/c, likewise cannot balance the observed non-back-to-back recoil.

The conclusion is unambiguous and we state it plainly: *within RealQM as it stands, the energy and momentum of beta decay cannot be balanced without a neutrino.* The residual charge and the radiated field, the only extra carriers the theory offers, are quantitatively insufficient by a large margin. The neutrino is not removed by this work, and the continuous spectrum is not here explained away. What RealQM changes is the *timing*: the decay is deterministic and its statistics emergent, whatever the final-state carriers turn out to be — the neutrino included.

5 Conclusion

Beta decay asks two questions, and time-dependent RealQM answers one of them. On *timing*, it replaces irreducible randomness with determinism: the decay is triggered by a phase coincidence among charge domains rotating at their computed energies, the escape time is fixed by the initial phases, and the exponential, memoryless half-life emerges as the ensemble statistics of that coincidence rather than as evidence of fundamental chance. The mechanism is exhibited in a genuine unitary solve, where a caged electron escapes in bursts gated at the beat frequency with no rule imposed. On *carriers*, we are equally clear and the result is negative: energy conservation holds by construction, but the residual recoil and the radiated field are quantitatively far too small — by about two orders of magnitude — to carry the majority share of energy and the momentum imbalance that the antineutrino accounts for, so the neutrino is not eliminated. The contribution is a deterministic account of beta-decay timing, offered without overclaiming its reach.

References

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